
pyLBM Documentation

Release 0.3.0

Benjamin Graille, Loïc Gouarin

Apr 12, 2017

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pyLBM is an all-in-one package for numerical simulations using Lattice Boltzmann solvers.

pyLBM is licensed under the BSD license, enabling reuse with few restrictions.

pyLBM can be a simple way to make numerical simulations by using the Lattice Boltzmann method.

To install pyLBM, you have several ways. You can install the last version on Pypi

```
pip install pyLBM
```

You can also clone the project

```
git clone https://github.com/pylbm/pylbm
```

and then use the commands

```
pip install -r requirements.txt
python setup.py install
```

or

```
pip install -r requirements.txt
python setup.py install --user
```

Once the package is installed you just have to understand how build a dictionary that will be understood by pyLBM to perform the simulation. The dictionary should contain all the needed informations as

- the geometry (see [here](#) for documentation)
- the scheme (see [here](#) for documentation)
- the boundary conditions (see [here](#) for documentation)
- another informations like the space step, the scheme velocity, the generator of the functions...

To understand how to use pyLBM, you have a lot of Python notebooks in the tutorial.

The Geometry of the simulation

With pyLBM, the numerical simulations can be performed in a domain with a complex geometry. This geometry is construct without considering a particular mesh but only with geometrical objects. All the geometrical informations are defined through a dictionary and put into an object of the class *Geometry*.

First, the domain is put into a box: a segment in 1D, a rectangle in 2D, and a rectangular parallelepiped in 3D.

Then, the domain is modified by adding or deleting some elementary shapes. In 2D, the elementary shapes are

- a *Circle*
- an *Ellipse*
- a *Parallelogram*
- a *Triangle*

From version 0.2, the geometrical elements are implemented in 3D. The elementary shapes are

- a *Sphere*
- an *Ellipsoid*
- a *Parallelepiped*
- a Cylinder with a 2D-base
 - *Cylinder (Circle)*
 - *Cylinder (Ellipse)*
 - *Cylinder (Triangle)*

Several examples of geometries can be found in `demo/examples/geometry/`

Examples in 1D

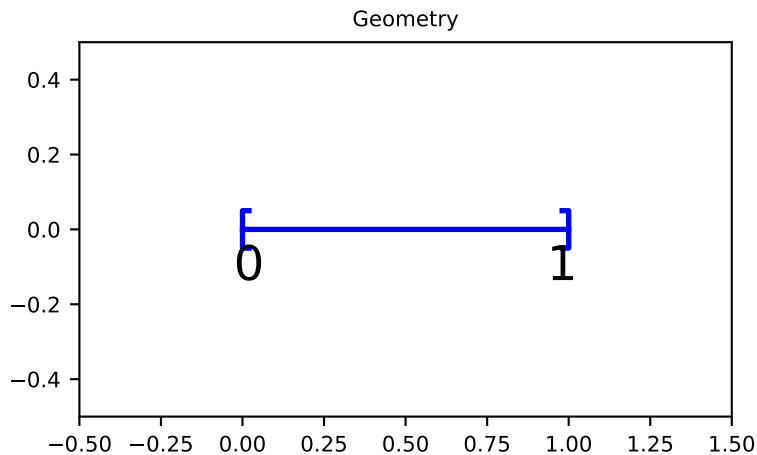
script

The segment $[0, 1]$

```
d = {'box':{'x': [0, 1], 'label': [0, 1]}}
g = pyLBM.Geometry(d)
g.visualize(viewlabel = True)
```

```
# Authors:
#   Loic Gouarin <loic.gouarin@math.u-psud.fr>
#   Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
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"""
Example of a 1D geometry: the segment  $[0, 1]$ 
"""
import pyLBM
d = {'box':{'x': [0, 1], 'label': [0, 1]}}
g = pyLBM.Geometry(d)
g.visualize(viewlabel = True)
```



The segment $[0, 1]$ is created by the dictionary with the key `box`. We then add the labels 0 and 1 on the edges with the key `label`. The result is then visualized with the labels by using the method `visualize`. If no labels are given in the dictionary, the default value is -1.

Examples in 2D

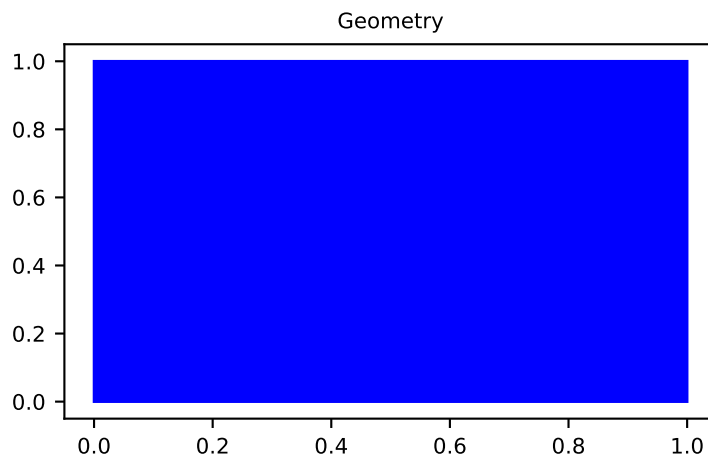
script

The square $[0, 1]^2$

```
d = {'box':{'x': [0, 1], 'y': [0, 1]}}
g = pyLBM.Geometry(d)
g.visualize()
```

```
# Authors:
#   Loic Gouarin <loic.gouarin@math.u-psud.fr>
#   Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
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"""
Example of a 2D geometry: the square  $[0, 1] \times [0, 1]$ 
"""
import pyLBM
d = {'box':{'x': [0, 1], 'y': [0, 1]}}
g = pyLBM.Geometry(d)
g.visualize()
```



The square $[0, 1]^2$ is created by the dictionary with the key `box`. The result is then visualized by using the method `visualize`.

We then add the labels on each edge of the square through a list of integers with the conventions:

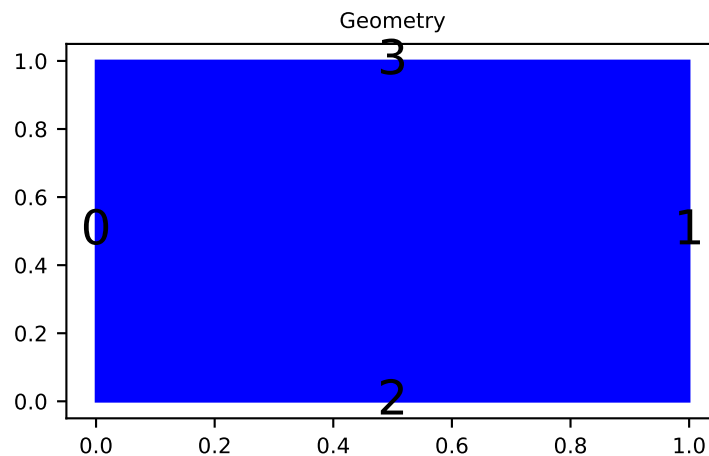
- first for the left ($x = x_{\min}$)
- third for the bottom ($y = y_{\min}$)
- second for the right ($x = x_{\max}$)
- fourth for the top ($y = y_{\max}$)

```
d = {'box':{'x': [0, 1], 'y': [0, 1], 'label':[0, 1, 2, 3]}}
g = pyLBM.Geometry(d)
g.visualize(viewlabel = True)
```

```
# Authors:
#   Loic Gouarin <loic.gouarin@math.u-psud.fr>
#   Benjamin Graille <benjamin.graille@math.u-psud.fr>
```

```
#
# License: BSD 3 clause

"""
Example of a 2D geometry: the square [0,1]x[0,1] with labels
"""
import pyLBM
d = {'box':{'x': [0, 1], 'y': [0, 1], 'label':[0, 1, 2, 3]}}
g = pyLBM.Geometry(d)
g.visualize(viewlabel = True)
```



If all the labels have the same value, a shorter solution is to give only the integer value of the label instead of the list. If no labels are given in the dictionary, the default value is -1.

script 3 script 2 script 1

A square with a hole

The unit square $[0, 1]^2$ can be holed with a circle (script 1) or with a triangular or with a parallelogram (script 3)

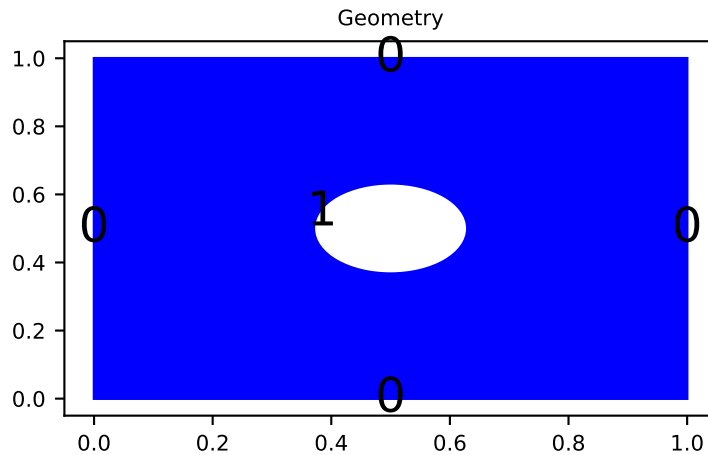
In the first example, a solid disc lies in the fluid domain defined by a *circle* with a center of (0.5, 0.5) and a radius of 0.125

```
d = {'box':{'x': [0, 1], 'y': [0, 1], 'label':0},
      'elements':[pyLBM.Circle((.5, .5), .125, label = 1)],
}
g = pyLBM.Geometry(d)
g.visualize(viewlabel=True)
```

```
# Authors:
#   Loic Gouarin <loic.gouarin@math.u-psud.fr>
#   Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
# License: BSD 3 clause

"""
Example of a 2D geometry: the square [0,1]x[0,1] with a circular hole
```

```
"""
import pyLBM
d = {'box':{'x': [0, 1], 'y': [0, 1], 'label':0},
      'elements':[pyLBM.Circle((.5, .5), .125, label = 1)],
}
g = pyLBM.Geometry(d)
g.visualize(viewlabel=True)
```



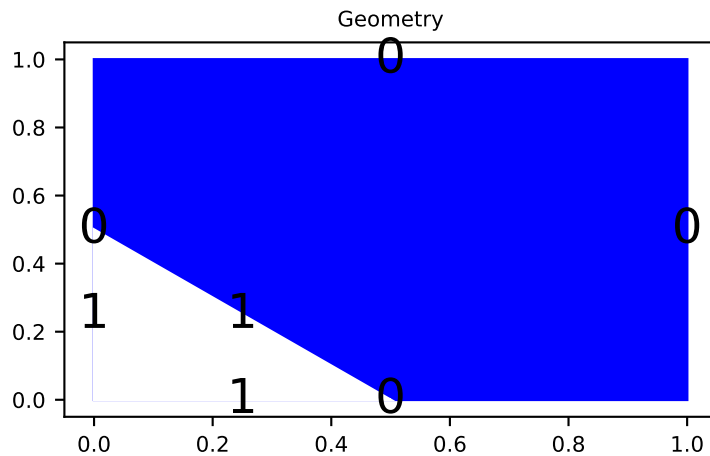
The dictionary of the geometry then contains an additional key `elements` that is a list of elements. In this example, the circle is labeled by 1 while the edges of the square by 0.

The element can be also a *triangle*

```
d = {'box':{'x': [0, 1], 'y': [0, 1], 'label':0},
      'elements':[pyLBM.Triangle((0.,0.), (0.,.5), (.5, 0.), label = 1)],
}
g = pyLBM.Geometry(d)
g.visualize(viewlabel=True)
```

```
# Authors:
#   Loic Gouarin <loic.gouarin@math.u-psud.fr>
#   Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
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"""
Example of a 2D geometry: the square [0,1]x[0,1] with a triangular hole
"""
import pyLBM
d = {'box':{'x': [0, 1], 'y': [0, 1], 'label':0},
      'elements':[pyLBM.Triangle((0.,0.), (0.,.5), (.5, 0.), label = 1)],
}
g = pyLBM.Geometry(d)
g.visualize(viewlabel=True)
```

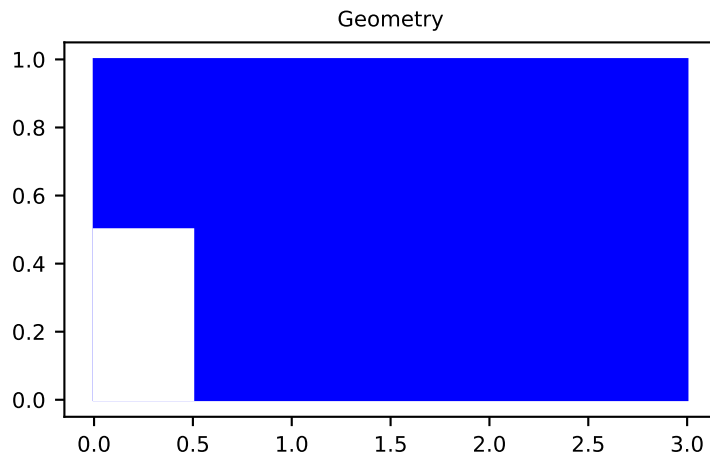


or a *parallelogram*

```
d = {'box':{'x': [0, 3], 'y': [0, 1], 'label':[1, 2, 0, 0]},
      'elements':[pyLBM.Parallelogram((0.,0.), (.5,0.), (0., .5), label = 0)],
    }
g = pyLBM.Geometry(d)
g.visualize()
```

```
# Authors:
#   Loic Gouarin <loic.gouarin@math.u-psud.fr>
#   Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
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"""
Example of a 2D geometry: the square [0,1]x[0,1] with a step
"""
import pyLBM
d = {'box':{'x': [0, 3], 'y': [0, 1], 'label':[1, 2, 0, 0]},
      'elements':[pyLBM.Parallelogram((0.,0.), (.5,0.), (0., .5), label = 0)],
    }
g = pyLBM.Geometry(d)
g.visualize()
```



script

A complex cavity

A complex geometry can be build by using a list of elements. In this example, the box is fixed to the unit square $[0, 1]^2$. A square hole is added with the argument `isfluid=False`. A strip and a circle are then added with the argument `isfluid=True`. Finally, a square hole is put. The value of `elements` contains the list of all the previous elements. Note that the order of the elements in the list is relevant.

```
square = pyLBM.Parallelogram((.1, .1), (.8, 0), (0, .8), isfluid=False)
strip = pyLBM.Parallelogram((0, .4), (1, 0), (0, .2), isfluid=True)
circle = pyLBM.Circle((.5, .5), .25, isfluid=True)
inner_square = pyLBM.Parallelogram((.4, .5), (.1, .1), (.1, -.1), isfluid=False)
d = {'box':{'x': [0, 1], 'y': [0, 1], 'label':0},
     'elements':[square, strip, circle, inner_square],
}
g = pyLBM.Geometry(d)
g.visualize()
```

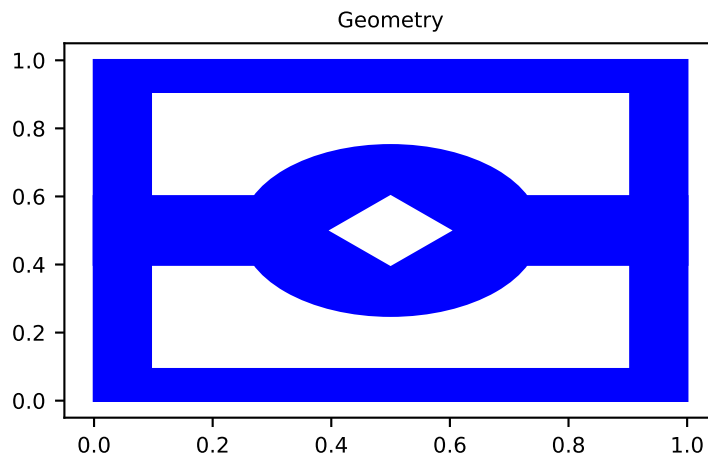
Once the geometry is built, it can be modified by adding or deleting other elements. For instance, the four corners of the cavity can be rounded in this way.

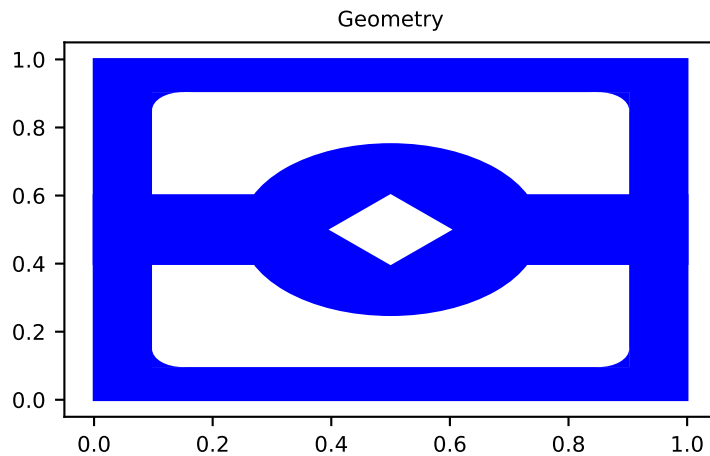
```
g.add_elem(pyLBM.Parallelogram((0.1, 0.9), (0.05, 0), (0, -0.05), isfluid=True))
g.add_elem(pyLBM.Circle((0.15, 0.85), 0.05, isfluid=False))
g.add_elem(pyLBM.Parallelogram((0.1, 0.1), (0.05, 0), (0, 0.05), isfluid=True))
g.add_elem(pyLBM.Circle((0.15, 0.15), 0.05, isfluid=False))
g.add_elem(pyLBM.Parallelogram((0.9, 0.9), (-0.05, 0), (0, -0.05), isfluid=True))
g.add_elem(pyLBM.Circle((0.85, 0.85), 0.05, isfluid=False))
g.add_elem(pyLBM.Parallelogram((0.9, 0.1), (-0.05, 0), (0, 0.05), isfluid=True))
g.add_elem(pyLBM.Circle((0.85, 0.15), 0.05, isfluid=False))
g.visualize()
```

```
# Authors:
#   Loic Gouarin <loic.gouarin@math.u-psud.fr>
#   Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
```

```
# License: BSD 3 clause

"""
Example of a complex geometry in 2D
"""
import pyLBM
square = pyLBM.Parallelogram((.1, .1), (.8, 0), (0, .8), isfluid=False)
strip = pyLBM.Parallelogram((0, .4), (1, 0), (0, .2), isfluid=True)
circle = pyLBM.Circle((.5, .5), .25, isfluid=True)
inner_square = pyLBM.Parallelogram((.4, .5), (.1, .1), (.1, -.1), isfluid=False)
d = {'box':{'x': [0, 1], 'y': [0, 1], 'label':0},
     'elements':[square, strip, circle, inner_square],
}
g = pyLBM.Geometry(d)
g.visualize()
# rounded inner angles
g.add_elem(pyLBM.Parallelogram((0.1, 0.9), (0.05, 0), (0, -0.05), isfluid=True))
g.add_elem(pyLBM.Circle((0.15, 0.85), 0.05, isfluid=False))
g.add_elem(pyLBM.Parallelogram((0.1, 0.1), (0.05, 0), (0, 0.05), isfluid=True))
g.add_elem(pyLBM.Circle((0.15, 0.15), 0.05, isfluid=False))
g.add_elem(pyLBM.Parallelogram((0.9, 0.9), (-0.05, 0), (0, -0.05), isfluid=True))
g.add_elem(pyLBM.Circle((0.85, 0.85), 0.05, isfluid=False))
g.add_elem(pyLBM.Parallelogram((0.9, 0.1), (-0.05, 0), (0, 0.05), isfluid=True))
g.add_elem(pyLBM.Circle((0.85, 0.15), 0.05, isfluid=False))
g.visualize()
```





Examples in 3D

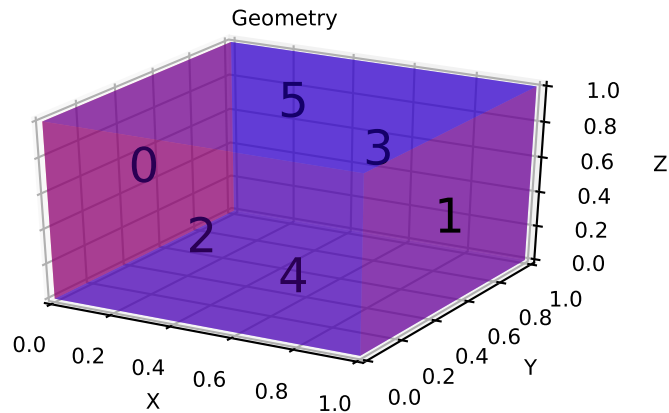
script

The cube $[0, 1]^3$

```
d = {'box':{'x': [0, 1], 'y': [0, 1], 'z':[0, 1], 'label':list(range(6))}}
g = pyLBM.Geometry(d)
g.visualize(viewlabel=True)
```

```
# Authors:
#   Loic Gouarin <loic.gouarin@math.u-psud.fr>
#   Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
# License: BSD 3 clause

"""
Example of a 3D geometry: the cube  $[0,1] \times [0,1] \times [0,1]$ 
"""
from six.moves import range
import pyLBM
d = {'box':{'x': [0, 1], 'y': [0, 1], 'z':[0, 1], 'label':list(range(6))}}
g = pyLBM.Geometry(d)
g.visualize(viewlabel=True)
```



The cube $[0, 1]^3$ is created by the dictionary with the key `box`. The result is then visualized by using the method `visualize`.

We then add the labels on each edge of the square through a list of integers with the conventions:

- first for the left ($x = x_{\min}$)
- third for the bottom ($y = y_{\min}$)
- fifth for the front ($z = z_{\min}$)
- second for the right ($x = x_{\max}$)
- fourth for the top ($y = y_{\max}$)
- sixth for the back ($z = z_{\max}$)

If all the labels have the same value, a shorter solution is to give only the integer value of the label instead of the list. If no labels are given in the dictionary, the default value is -1.

The cube $[0, 1]^3$ with a hole

```
d = {
    'box': {'x': [0, 1], 'y': [0, 1], 'z': [0, 1], 'label': 0},
    'elements': [pyLBM.Sphere((.5, .5, .5), .25, label=1)],
}
g = pyLBM.Geometry(d)
g.visualize(viewlabel=True)
```

```
# Authors:
#     Loic Gouarin <loic.gouarin@math.u-psud.fr>
#     Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
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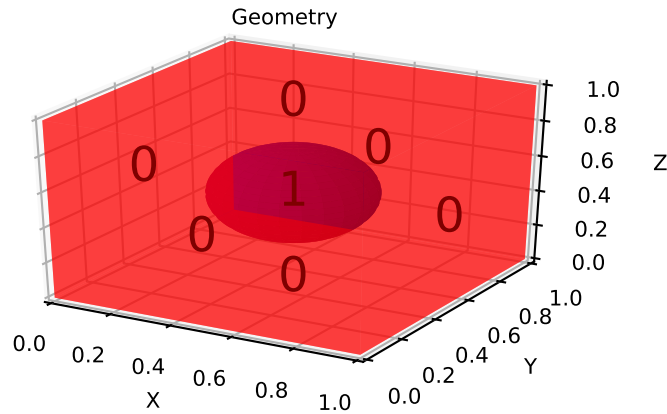
"""
Example of a 3D geometry: the cube  $[0, 1] \times [0, 1] \times [0, 1]$ 
"""
import pyLBM
d = {
    'box': {'x': [0, 1], 'y': [0, 1], 'z': [0, 1], 'label': 0},
    'elements': [pyLBM.Sphere((.5, .5, .5), .25, label=1)],
```



```

}
g = pyLBM.Geometry(d)
g.visualize(viewlabel=True)

```



The cube $[0, 1]^3$ and the spherical hole are created by the dictionary with the keys `box` and `elements`. The result is then visualized by using the method `visualize`.

The Domain of the simulation

With pyLBM, the numerical simulations can be performed in a domain with a complex geometry. The creation of the geometry from a dictionary is explained here. All the informations needed to build the domain are defined through a dictionary and put in a object of the class `Domain`.

The domain is built from three types of informations:

- a geometry (class `Geometry`),
- a stencil (class `Stencil`),
- a space step (a float for the grid step of the simulation).

The domain is a uniform cartesian discretization of the geometry with a grid step dx . The whole box is discretized even if some elements are added to reduce the domain of the computation. The stencil is necessary in order to know the maximal velocity in each direction so that the corresponding number of phantom cells are added at the borders of the domain (for the treatment of the boundary conditions). The user can get the coordinates of the points in the domain by the fields `x`, `y`, and `z`. By convention, if the spatial dimension is one, `y=z=None`; and if it is two, `z=None`.

Several examples of domains can be found in `demo/examples/domain/`

Examples in 1D

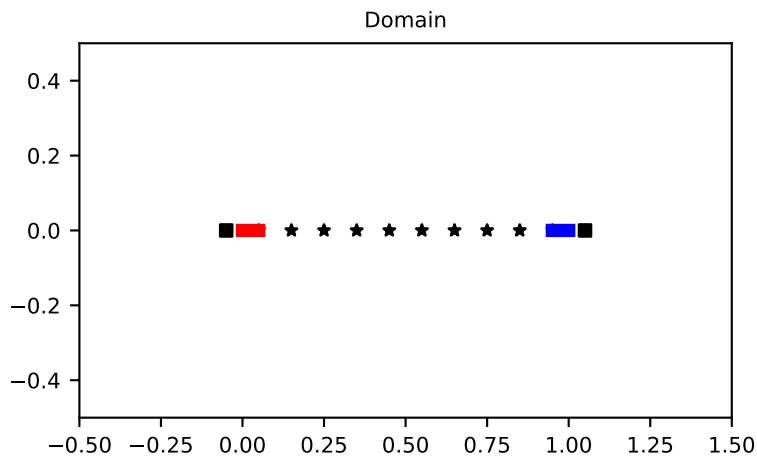
script

The segment $[0, 1]$ with a D_1Q_3

```
dico = {
    'box': {'x': [0, 1], 'label': 0},
    'space_step': 0.1,
    'schemes': [{'velocities': list(range(3))}],
}
dom = pyLBM.Domain(dico)
dom.visualize()
```

```
# Authors:
#     Loic Gouarin <loic.gouarin@math.u-psud.fr>
#     Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
# License: BSD 3 clause

"""
Example of a segment in 1D with a D1Q3
"""
from six.moves import range
import pyLBM
dico = {
    'box': {'x': [0, 1], 'label': 0},
    'space_step': 0.1,
    'schemes': [{'velocities': list(range(3))}],
}
dom = pyLBM.Domain(dico)
dom.visualize()
```



The segment $[0, 1]$ is created by the dictionary with the key `box`. The stencil is composed by the velocity $v_0 = 0$, $v_1 = 1$, and $v_2 = -1$. One phantom cell is then added at the left and at the right of the domain. The space step dx is taken to 0.1 to allow the visualization. The result is then visualized with the distance of the boundary points by using the method `visualize`.

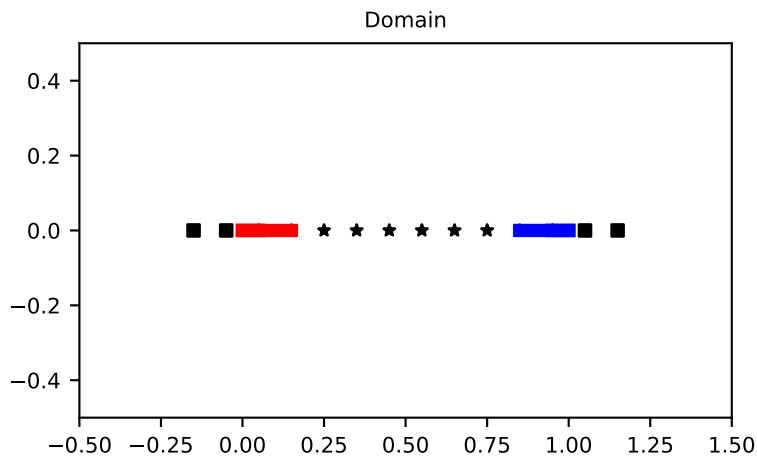
script

The segment $[0, 1]$ with a D_1Q_5

```
dico = {
    'box': {'x': [0, 1], 'label': 0},
    'space_step': 0.1,
    'schemes': [{'velocities': list(range(5))}],
}
dom = pyLBM.Domain(dico)
dom.visualize()
```

```
# Authors:
#     Loic Gouarin <loic.gouarin@math.u-psud.fr>
#     Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
# License: BSD 3 clause

"""
Example of a segment in 1D with a D1Q5
"""
from six.moves import range
import pyLBM
dico = {
    'box': {'x': [0, 1], 'label': 0},
    'space_step': 0.1,
    'schemes': [{'velocities': list(range(5))}],
}
dom = pyLBM.Domain(dico)
dom.visualize()
```



The segment $[0, 1]$ is created by the dictionary with the key `box`. The stencil is composed by the velocity $v_0 = 0$, $v_1 = 1$, $v_2 = -1$, $v_3 = 2$, $v_4 = -2$. Two phantom cells are then added at the left and at the right of the domain. The space step dx is taken to 0.1 to allow the visualization. The result is then visualized with the distance of the boundary points by using the method `visualize`.

Examples in 2D

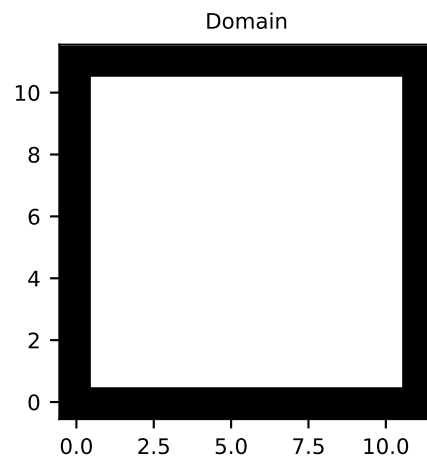
script

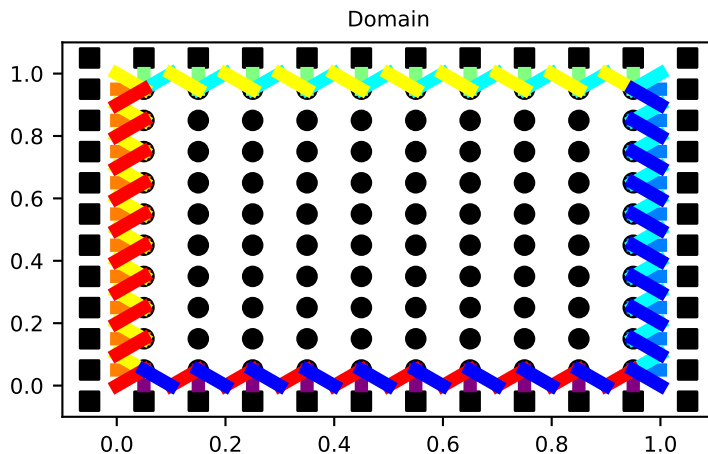
The square $[0, 1]^2$ with a D_2Q_9

```
dico = {
    'box': {'x': [0, 1], 'y': [0, 1], 'label': 0},
    'space_step': 0.1,
    'schemes': [{'velocities': list(range(9))}],
}
dom = pyLBM.Domain(dico)
dom.visualize()
dom.visualize(view_distance=True)
```

```
# Authors:
#     Loic Gouarin <loic.gouarin@math.u-psud.fr>
#     Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
# License: BSD 3 clause

"""
Example of a square in 2D with a D2Q9
"""
from six.moves import range
import pyLBM
dico = {
    'box': {'x': [0, 1], 'y': [0, 1], 'label': 0},
    'space_step': 0.1,
    'schemes': [{'velocities': list(range(9))}],
}
dom = pyLBM.Domain(dico)
dom.visualize()
dom.visualize(view_distance=True)
```





The square $[0, 1]^2$ is created by the dictionary with the key `box`. The stencil is composed by the nine velocities

$$\begin{aligned} v_0 &= (0, 0), \\ v_1 &= (1, 0), v_2 = (0, 1), v_3 = (-1, 0), v_4 = (0, -1), \\ v_5 &= (1, 1), v_6 = (-1, 1), v_7 = (-1, -1), v_8 = (1, -1). \end{aligned} \quad (1.1)$$

One phantom cell is then added all around the square. The space step dx is taken to 0.1 to allow the visualization. The result is then visualized by using the method `visualize`. This method can be used without parameter: the domain is visualize in white for the fluid part (where the computation is done) and in black for the solid part (the phantom cells or the obstacles). An optional parameter `view_distance` can be used to visualize more precisely the points (a black circle inside the domain and a square outside). Color lines are added to visualize the position of the border: for each point that can reach the border for a given velocity in one time step, the distance to the border is computed.

script 1

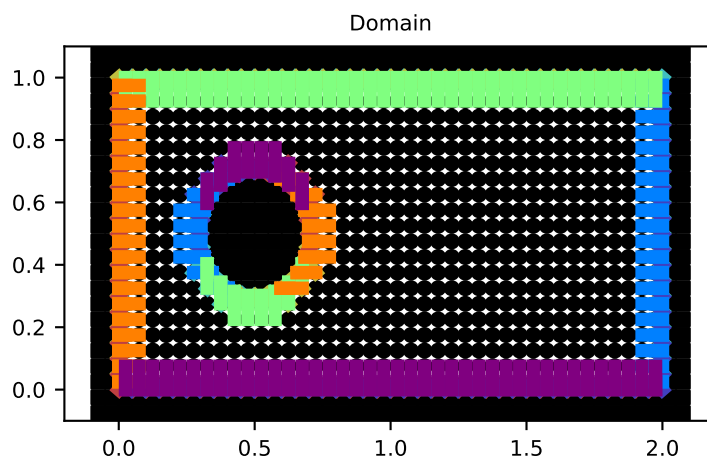
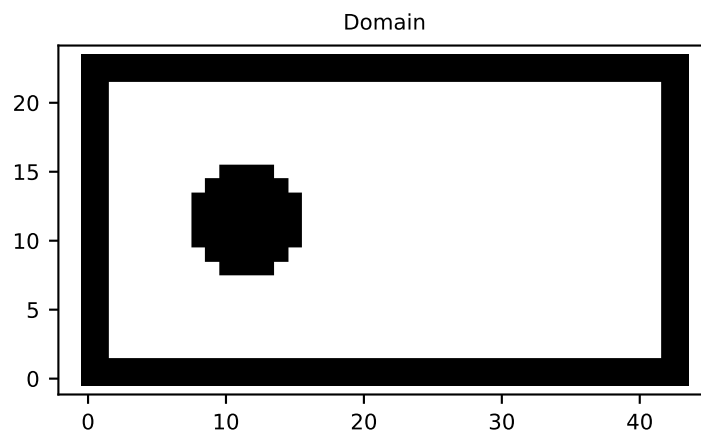
A square with a hole with a D_2Q_{13}

The unit square $[0, 1]^2$ can be holed with a circle. In this example, a solid disc lies in the fluid domain defined by a `circle` with a center of (0.5, 0.5) and a radius of 0.125

```
dico = {
    'box': {'x': [0, 2], 'y': [0, 1], 'label': 0},
    'elements': [pyLBM.Circle((0.5, 0.5), 0.2)],
    'space_step': 0.05,
    'schemes': [{'velocities': list(range(13))}],
}
dom = pyLBM.Domain(dico)
dom.visualize()
dom.visualize(view_distance=True)
```

```
# Authors:
#   Loic Gouarin <loic.gouarin@math.u-psud.fr>
#   Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
# License: BSD 3 clause
```

```
"""
Example of a square in 2D with a circular hole with a D2Q13
"""
from six.moves import range
import pyLBM
dico = {
    'box':{'x': [0, 2], 'y': [0, 1], 'label':0},
    'elements':[pyLBM.Circle((0.5,0.5), 0.2)],
    'space_step':0.05,
    'schemes':[{'velocities':list(range(13))}],
}
dom = pyLBM.Domain(dico)
dom.visualize()
dom.visualize(view_distance=True)
```



script

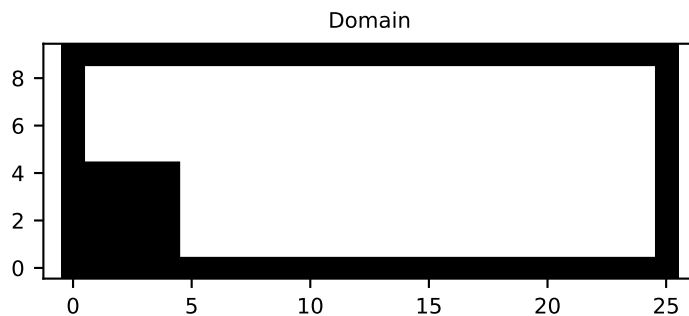
A step with a D_2Q_9

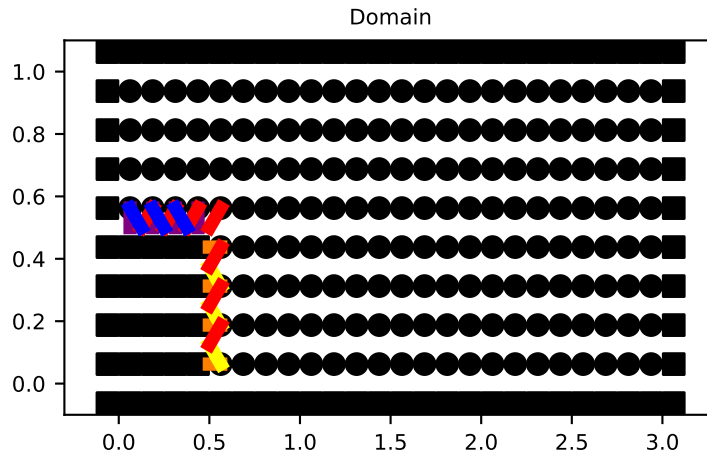
A step can be build by removing a rectangle in the left corner. For a D_2Q_9 , it gives the following domain.

```
dico = {
    'box': {'x': [0, 3], 'y': [0, 1], 'label': 0},
    'elements': [pyLBM.Parallelogram((0., 0.), (.5, 0.), (0., .5), label=1)],
    'space_step': 0.125,
    'schemes': [{'velocities': list(range(9))}],
}
dom = pyLBM.Domain(dico)
dom.visualize()
dom.visualize(view_distance=True, label=1)
```

```
# Authors:
#     Loic Gouarin <loic.gouarin@math.u-psud.fr>
#     Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
# License: BSD 3 clause

"""
Example of the backward facing step in 2D with a D2Q9
"""
from six.moves import range
import pyLBM
dico = {
    'box': {'x': [0, 3], 'y': [0, 1], 'label': 0},
    'elements': [pyLBM.Parallelogram((0., 0.), (.5, 0.), (0., .5), label=1)],
    'space_step': 0.125,
    'schemes': [{'velocities': list(range(9))}],
}
dom = pyLBM.Domain(dico)
dom.visualize()
dom.visualize(view_distance=True, label=1)
```





Note that the distance with the bound is visible only for the specified labels.

Examples in 3D

script

The cube $[0, 1]^3$ with a D_3Q_{19}

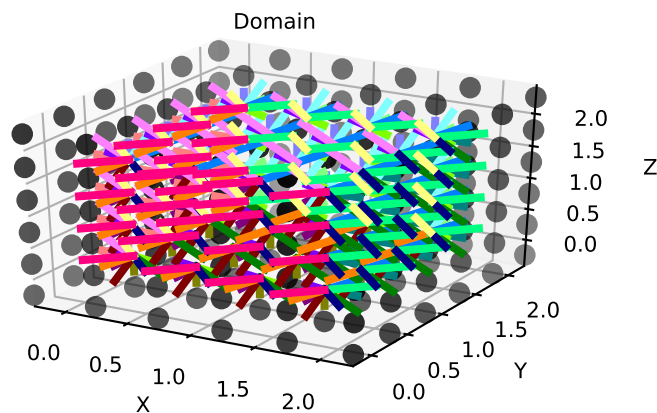
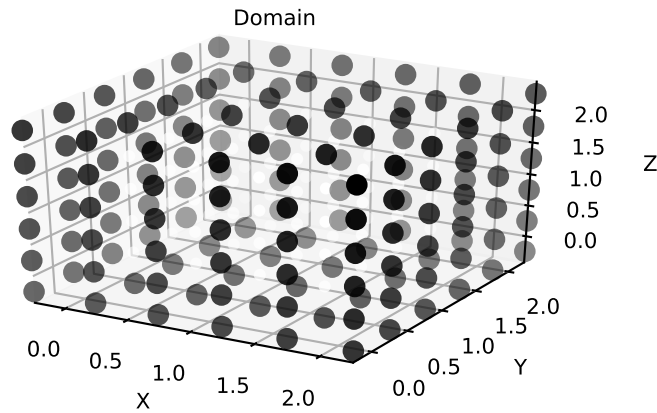
```
dico = {
    'box': {'x': [0, 2], 'y': [0, 2], 'z': [0, 2], 'label': 0},
    'space_step': .5,
    'schemes': [{'velocities': list(range(19))}]
}
dom = pyLBM.Domain(dico)
dom.visualize()
dom.visualize(view_distance=True)
```

```
# Authors:
#     Loic Gouarin <loic.gouarin@math.u-psud.fr>
#     Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
# License: BSD 3 clause

"""
Example of the cube in 3D with a D3Q19
"""
from six.moves import range
import pyLBM
dico = {
    'box': {'x': [0, 2], 'y': [0, 2], 'z': [0, 2], 'label': 0},
    'space_step': .5,
    'schemes': [{'velocities': list(range(19))}]
}
dom = pyLBM.Domain(dico)
```



```
dom.visualize()
dom.visualize(view_distance=True)
```



The cube $[0, 1]^3$ is created by the dictionary with the key `box` and the first 19th velocities. The result is then visualized by using the method `visualize`.

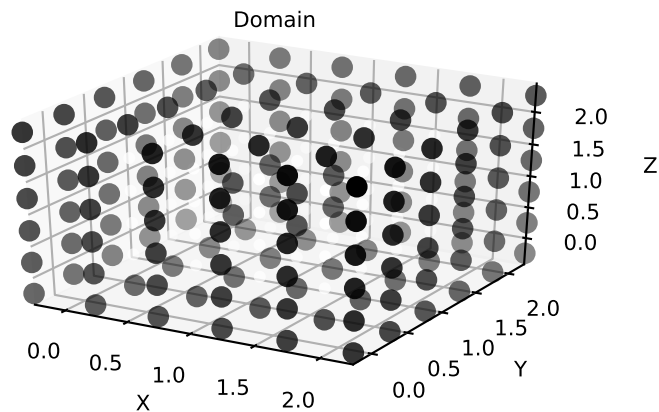
The cube with a hole with a D_3Q_{19}

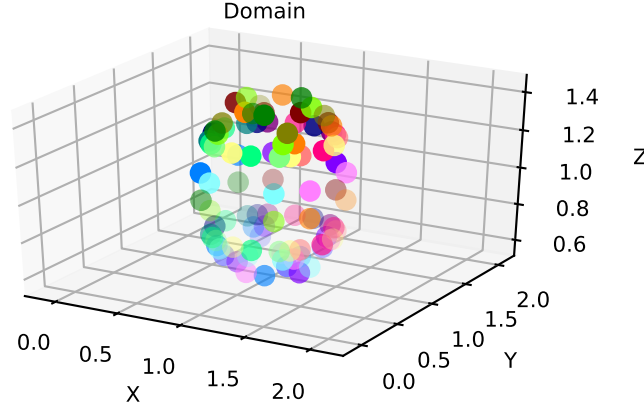
```
dico = {
    'box':{'x': [0, 2], 'y': [0, 2], 'z':[0, 2], 'label':0},
    'elements':[pyLBM.Sphere((1,1,1), 0.5, label = 1)],
    'space_step':.5,
    'schemes': [{'velocities':list(range(19))}]
}
dom = pyLBM.Domain(dico)
```

```
dom.visualize()
dom.visualize(view_distance=False, view_bound=True, label=1, view_in=False, view_
↪out=False)
```

```
# Authors:
#   Loic Gouarin <loic.gouarin@math.u-psud.fr>
#   Benjamin Graille <benjamin.graille@math.u-psud.fr>
#
# License: BSD 3 clause

"""
Example of the cube in 3D with a D3Q19
"""
from six.moves import range
import pyLBM
dico = {
    'box':{'x': [0, 2], 'y': [0, 2], 'z':[0, 2], 'label':0},
    'elements':[pyLBM.Sphere((1,1,1), 0.5, label = 1)],
    'space_step':.5,
    'schemes':[{'velocities':list(range(19))}]
}
dom = pyLBM.Domain(dico)
dom.visualize()
dom.visualize(view_distance=False, view_bound=True, label=1, view_in=False, view_
↪out=False)
```





The Scheme

With pyLBM, elementary schemes can be gathered and coupled through the equilibrium in order to simplify the implementation of the vectorial schemes. Of course, the user can implement a single elementary scheme and then recover the classical framework of the d’Humières schemes.

For pyLBM, the `scheme` is performed through a dictionary. The generalized d’Humières framework for vectorial schemes is used [dH92], [G14]. In the first section, we describe how build an elementary scheme. Then the vectorial schemes are introduced as coupled elementary schemes.

The elementary schemes

Let us first consider a regular lattice L in dimension d with a typical mesh size dx , and the time step dt . The scheme velocity λ is then defined by $\lambda = dx/dt$. We introduce a set of q velocities adapted to this lattice $\{v_0, \dots, v_{q-1}\}$, that is to say that, if x is a point of the lattice L , the point $x + v_j dt$ is on the lattice for every $j \in \{0, \dots, q-1\}$.

The aim of the $DdQq$ scheme is to compute a distribution function vector $\mathbf{f} = (f_0, \dots, f_{q-1})$ on the lattice L at discret values of time. The scheme splits into two phases: the relaxation and the transport. That is, the passage from the time t to the time $t + dt$ consists in the succession of these two phases.

- the relaxation phase

This phase, also called collision, is local in space: on every site x of the lattice, the values of the vector \mathbf{f} are modified, the result after the collision being denoted by \mathbf{f}^* . The operator of collision is a linear operator of relaxation toward an equilibrium value denoted \mathbf{f}^{eq} .

pyLBM uses the framework of d’Humières: the linear operator of the collision is diagonal in a special basis called moments denoted by $\mathbf{m} = (m_0, \dots, m_{q-1})$. The change-of-basis matrix M is such that $\mathbf{m} = M\mathbf{f}$ and $\mathbf{f} = M^{-1}\mathbf{m}$. In the basis of the moments, the collision operator then just reads

$$m_k^* = m_k - s_k(m_k - m_k^{\text{eq}}), \quad 0 \leq k \leq q-1,$$

where s_k is the relaxation parameter associated to the k th moment. The k th moment is said conserved during the collision if the associated relaxation parameter $s_k = 0$.

By analogy with the kinetic theory, the change-of-basis matrix M is defined by a set of polynomials with d variables (P_0, \dots, P_{q-1}) by

$$M_{ij} = P_i(v_j).$$

- the transport phase

This phase just consists in a shift of the indices and reads

$$f_j(x, t + dt) = f_j^*(x - v_j dt, t), \quad 0 \leq j \leq q-1,$$

Notations

The `scheme` is defined and build through a dictionary in pyLBM. Let us first list the several key words of this dictionary:

- `dim`: the spatial dimension. This argument is optional if the geometry is known, that is if the dimension can be computed through the list of the variables;
- `scheme_velocity`: the velocity of the scheme denoted by λ in the previous section and defined as the spatial step over the time step ($\lambda = dx/dt$);
- `schemes`: the list of the schemes. In pyLBM, several coupled schemes can be used, the coupling being done through the equilibrium values of the moments. Some examples with only one scheme and with more than one schemes are given in the next sections. Each element of the list should be a dictionary with the following key words:
 - `velocities`: the list of the velocity indices,
 - `conserved_moments`: the list of the conserved moments (list of symbolic variables),
 - `polynomials`: the list of the polynomials that define the moments, the polynomials are built with the symbolic variables X, Y, and Z,
 - `equilibrium`: the list of the equilibrium value of the moments,
 - `relaxation_parameters`: the list of the relaxation parameters, (by convention, the relaxation parameter of a conserved moment is taken to 0).

Examples in 1D

script

D_1Q_2 for the advection

A velocity $c \in \mathbb{R}$ being given, the advection equation reads

$$\partial_t u(t, x) + c \partial_x u(t, x) = 0, \quad t > 0, x \in \mathbb{R}.$$

Taken for instance $c = 0.5$, the following scheme can be used:

```
import sympy as sp
import pyLBM
u, X = sp.symbols('u, X')

d = {
    'dim':1,
    'scheme_velocity':1.,
```

```

'schemes':[
    {
        'velocities': [1, 2],
        'conserved_moments':u,
        'polynomials': [1, X],
        'equilibrium': [u, .5*u],
        'relaxation_parameters': [0., 1.9],
    },
],
}
s = pyLBM.Scheme(d)
print(s)

```

The dictionary `d` is used to set the dimension to 1, the scheme velocity to 1. The used scheme has two velocities: the first one $v_0 = 1$ and the second one $v_1 = -1$. The polynomials that define the moments are $P_0 = 1$ and $P_1 = X$ so that the matrix of the moments is

$$M = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

with the convention $M_{ij} = P_i(v_j)$. Then, there are two distribution functions f_0 and f_1 that move at the velocities v_0 and v_1 , and two moments $m_0 = f_0 + f_1$ and $m_1 = f_0 - f_1$. The first moment m_0 is conserved during the relaxation phase (as the associated relaxation parameter is set to 0), while the second moment m_1 relaxes to its equilibrium value $0.5m_0$ with a relaxation parameter 1.9 by the relation

$$m_1^* = m_1 - 1.9(m_1 - 0.5m_0).$$

script

D_1Q_2 for Burger's

The Burger's equation reads

$$\partial_t u(t, x) + \frac{1}{2} \partial_x u^2(t, x) = 0, \quad t > 0, x \in \mathbb{R}.$$

The following scheme can be used:

```

import sympy as sp
import pyLBM
u, X = sp.symbols('u, X')

d = {
    'dim':1,
    'scheme_velocity':1.,
    'schemes':[
        {
            'velocities': [1, 2],
            'conserved_moments':u,
            'polynomials': [1, X],
            'equilibrium': [u, .5*u**2],
            'relaxation_parameters': [0., 1.9],
        },
    ],
}
s = pyLBM.Scheme(d)
print(s)

```

The same dictionary has been used for this application with only one modification: the equilibrium value of the second moment m_1^{eq} is taken to $\frac{1}{2}m_0^2$.

script

D_1Q_3 for the wave equation

The wave equation is rewritten into the system of two partial differential equations

$$\begin{cases} \partial_t u(t, x) + \partial_x v(t, x) = 0, & t > 0, x \in \mathbb{R}, \\ \partial_t v(t, x) + c^2 \partial_x u(t, x) = 0, & t > 0, x \in \mathbb{R}. \end{cases}$$

The following scheme can be used:

```
import sympy as sp
import pyLBM
u, v, X = sp.symbols('u, v, X')

c = 0.5
d = {
    'dim':1,
    'scheme_velocity':1.,
    'schemes':[{
        'velocities': [0, 1, 2],
        'conserved_moments':[u, v],
        'polynomials': [1, X, 0.5*X**2],
        'equilibrium': [u, v, .5*c**2*u],
        'relaxation_parameters': [0., 0., 1.9],
    }],
}
s = pyLBM.Scheme(d)
print(s)
```

Examples in 2D

script

D_2Q_4 for the advection

A velocity $(c_x, c_y) \in \mathbb{R}^2$ being given, the advection equation reads

$$\partial_t u(t, x, y) + c_x \partial_x u(t, x, y) + c_y \partial_y u(t, x, y) = 0, \quad t > 0, x, y \in \mathbb{R}.$$

Taken for instance $c_x = 0.1, c_y = 0.2$, the following scheme can be used:

```
import sympy as sp
import pyLBM
u, X, Y = sp.symbols('u, X, Y')

d = {
    'dim':2,
    'scheme_velocity':1.,
    'schemes':[{
        'velocities': [1, 2, 3, 4],
        'conserved_moments':u,
        'polynomials': [1, X, Y, X**2-Y**2],
        'equilibrium': [u, .1*u, .2*u, 0.],
        'relaxation_parameters': [0., 1.9, 1.9, 1.4],
    }],
}
```

```

    ],
}
s = pyLBM.Scheme(d)
print(s)

```

The dictionary `d` is used to set the dimension to 2, the scheme velocity to 1. The used scheme has four velocities: $v_0 = (1, 0)$, $v_1 = (0, 1)$, $v_2 = (-1, 0)$, and $v_3 = (0, -1)$. The polynomials that define the moments are $P_0 = 1$, $P_1 = X$, $P_2 = Y$, and $P_3 = X^2 - Y^2$ so that the matrix of the moments is

$$M = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$$

with the convention $M_{ij} = P_i(v_j)$. Then, there are four distribution functions $f_j, 0 \leq j \leq 3$ that move at the velocities v_j , and four moments $m_k = \sum_{j=0}^3 M_{kj} f_j$. The first moment m_0 is conserved during the relaxation phase (as the associated relaxation parameter is set to 0), while the other moments $m_k, 1 \leq k \leq 3$ relax to their equilibrium values by the relations

$$\begin{cases} m_1^* = m_1 - 1.9(m_1 - 0.1m_0), \\ m_2^* = m_2 - 1.9(m_2 - 0.2m_0), \\ m_3^* = (1 - 1.4)m_3. \end{cases}$$

script

D_2Q_9 for Navier-Stokes

The system of the compressible Navier-Stokes equations reads

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0, \\ \partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p = \kappa \nabla (\nabla \cdot \mathbf{u}) + \eta \nabla \cdot (\nabla \mathbf{u} + (\nabla \mathbf{u})^T - \nabla \cdot \mathbf{u} \mathbb{I}), \end{cases}$$

where we removed the dependency of all unknown on the variables (t, x, y) . The vector \mathbf{x} stands for (x, y) and, if ψ is a scalar function of \mathbf{x} and $\phi = (\phi_x, \phi_y)$ is a vectorial function of \mathbf{x} , the usual partial differential operators read

$$\begin{aligned} \nabla \psi &= (\partial_x \psi, \partial_y \psi), \\ \nabla \cdot \phi &= \partial_x \phi_x + \partial_y \phi_y, \\ \nabla \cdot (\phi \otimes \phi) &= (\nabla \cdot (\phi_x \phi), \nabla \cdot (\phi_y \phi)). \end{aligned}$$

The coefficients κ and η are the bulk and the shear viscosities.

The following dictionary can be used to simulate the system of Navier-Stokes equations in the limit of small velocities:

```

from six.moves import range
import sympy as sp
import pyLBM
rho, qx, qy, X, Y = sp.symbols('rho, qx, qy, X, Y')

dx = 1./256    # space step
eta = 1.25e-5   # shear viscosity
kappa = 10*eta # bulk viscosity
sb = 1./(.5+kappa*3./dx)
ss = 1./(.5+eta*3./dx)
d = {
    'dim':2,
    'scheme_velocity':1.,

```

```

'schemes':[{
    'velocities':list(range(9)),
    'conserved_moments':[rho, qx, qy],
    'polynomials':[
        1, X, Y,
        3*(X**2+Y**2)-4,
        (9*(X**2+Y**2)**2-21*(X**2+Y**2)+8)/2,
        3*X*(X**2+Y**2)-5*X, 3*Y*(X**2+Y**2)-5*Y,
        X**2-Y**2, X*Y
    ],
    'relaxation_parameters':[0., 0., 0., sb, sb, sb, sb, ss, ss],
    'equilibrium':[
        rho, qx, qy,
        -2*rho + 3*qx**2 + 3*qy**2,
        rho + 3/2*qx**2 + 3/2*qy**2,
        -qx, -qy,
        qx**2 - qy**2, qx*qy
    ],
},],
}
s = pyLBM.Scheme(d)
print(s)

```

The scheme generated by the dictionary is the 9 velocities scheme with orthogonal moments introduced in [QdHL92]

Examples in 3D

script

D_3Q_6 for the advection

A velocity $(c_x, c_y, c_z) \in \mathbb{R}^3$ being given, the advection equation reads

$$\partial_t u(t, x, y, z) + c_x \partial_x u(t, x, y, z) + c_y \partial_y u(t, x, y, z) + c_z \partial_z u(t, x, y, z) = 0, \quad t > 0, x, y, z \in \mathbb{R}.$$

Taken for instance $c_x = 0.1, c_y = -0.1, c_z = 0.2$, the following scheme can be used:

```

from six.moves import range
import sympy as sp
import pyLBM
u, X, Y, Z = sp.symbols('u, X, Y, Z')

cx, cy, cz = .1, -.1, .2
d = {
    'dim':3,
    'scheme_velocity':1.,
    'schemes':[{
        'velocities': list(range(1,7)),
        'conserved_moments':u,
        'polynomials': [1, X, Y, Z, X**2-Y**2, X**2-Z**2],
        'equilibrium': [u, cx*u, cy*u, cz*u, 0., 0.],
        'relaxation_parameters': [0., 1.5, 1.5, 1.5, 1.5, 1.5],
    },],
}
s = pyLBM.Scheme(d)
print(s)

```


The dictionary `d` is used to set the dimension to 3, the scheme velocity to 1. The used scheme has six velocities: $v_0 = (0, 0, 1)$, $v_1 = (0, 0, -1)$, $v_2 = (0, 1, 0)$, $v_3 = (0, -1, 0)$, $v_4 = (1, 0, 0)$, and $v_5 = (-1, 0, 0)$. The polynomials that define the moments are $P_0 = 1$, $P_1 = X$, $P_2 = Y$, $P_3 = Z$, $P_4 = X^2 - Y^2$, and $P_5 = X^2 - Z^2$ so that the matrix of the moments is

$$M = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 1 \\ -1 & -1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

with the convention $M_{ij} = P_i(v_j)$. Then, there are six distribution functions f_j , $0 \leq j \leq 5$ that move at the velocities v_j , and six moments $m_k = \sum_{j=0}^5 M_{kj} f_j$. The first moment m_0 is conserved during the relaxation phase (as the associated relaxation parameter is set to 0), while the other moments m_k , $1 \leq k \leq 5$ relax to their equilibrium values by the relations

$$\begin{cases} m_1^* = m_1 - 1.5(m_1 - 0.1m_0), \\ m_2^* = m_2 - 1.5(m_2 + 0.1m_0), \\ m_3^* = m_3 - 1.5(m_3 - 0.2m_0), \\ m_4^* = (1 - 1.5)m_4, \\ m_5^* = (1 - 1.5)m_5. \end{cases}$$

The vectorial schemes

With pyLBM, vectorial schemes can be built easily by using a list of elementary schemes. Each elementary scheme is given by a dictionary as in the previous section. The conserved moments of all the elementary schemes can be used in the equilibrium values of the non conserved moments, in order to couple the schemes. For more details on the vectorial schemes, the reader can refer to [GI4].

Examples in 1D

script

$D_1Q_{2,2}$ for the shallow water equation

A constant $g \in \mathbb{R}$ being given, the shallow water system reads

$$\begin{aligned} \partial_t h(t, x) + \partial_x q(t, x) &= 0, & t > 0, x \in \mathbb{R}, \\ \partial_t q(t, x) + \partial_x (q^2(t, x)/h(t, x) + gh^2(t, x)/2) &= 0, & t > 0, x \in \mathbb{R}. \end{aligned}$$

Taken for instance $g = 1$, the following scheme can be used:

```
import sympy as sp
import pyLBM

# parameters
h, q, X, LA, g = sp.symbols('h, q, X, LA, g')
la = 2. # velocity of the scheme
s_h, s_q = 1.7, 1.5 # relaxation parameters

d = {
    'dim': 1,
    'scheme_velocity': la,
```

```

'schemes':[
    {
        'velocities': [1, 2],
        'conserved_moments': h,
        'polynomials': [1, LA*X],
        'relaxation_parameters': [0, s_h],
        'equilibrium': [h, q],
    },
    {
        'velocities': [1, 2],
        'conserved_moments': q,
        'polynomials': [1, LA*X],
        'relaxation_parameters': [0, s_q],
        'equilibrium': [q, q**2/h+.5*g*h**2],
    },
],
'parameters': {LA: la, g: 1.},
}
s = pyLBM.Scheme(d)
print(s)

```

Two elementary schemes have been built, these two schemes are identical except for the equilibrium values of the non conserved moment and of the relaxation parameter: The first one is used to simulate the equation on h and the second one to simulate the equation on q . For each scheme, the equilibrium value of the non conserved moment is equal to the flux of the corresponding equation: the equilibrium value of the k th scheme can so depend on all the conserved moments (and not only on those of the k th scheme).

Examples in 2D

script

$D_2Q_{4,4,4}$ for the shallow water equation

A constant $g \in \mathbb{R}$ being given, the shallow water system reads

$$\begin{aligned}
 \partial_t h(t, x, y) + \partial_x q_x(t, x, y) + \partial_y q_y(t, x, y) &= 0, & t > 0, x, y \in \mathbb{R}, \\
 \partial_t q_x(t, x, y) + \partial_x (q_x^2(t, x, y)/h(t, x, y) + gh^2(t, x, y)/2) \\
 + \partial_y (q_x(t, x, y)q_y(t, x, y)/h(t, x, y)) &= 0, & t > 0, x, y \in \mathbb{R}, \\
 \partial_t q_y(t, x, y) + \partial_x (q_x(t, x, y)q_y(t, x, y)/h(t, x, y)) \\
 + \partial_y (q_y^2(t, x, y)/h(t, x, y) + gh^2(t, x, y)/2) &= 0, & t > 0, x, y \in \mathbb{R}.
 \end{aligned}$$

Taken for instance $g = 1$, the following scheme can be used:

```

import sympy as sp
import pyLBM

X, Y, LA, g = sp.symbols('X, Y, LA, g')
h, qx, qy = sp.symbols('h, qx, qy')

# parameters
la = 4 # velocity of the scheme
s_h = [0., 2., 2., 1.5]
s_q = [0., 1.5, 1.5, 1.2]

```

```

vitesse = [1,2,3,4]
polynomes = [1, LA*X, LA*Y, X**2-Y**2]

d = {
    'dim': 2,
    'scheme_velocity': la,
    'schemes': [
        {
            'velocities': vitesse,
            'conserved_moments': h,
            'polynomials': polynomes,
            'relaxation_parameters': s_h,
            'equilibrium': [h, qx, qy, 0.],
        },
        {
            'velocities': vitesse,
            'conserved_moments': qx,
            'polynomials': polynomes,
            'relaxation_parameters': s_q,
            'equilibrium': [qx, qx**2/h + 0.5*g*h**2, qx*qy/h, 0.],
        },
        {
            'velocities': vitesse,
            'conserved_moments': qy,
            'polynomials': polynomes,
            'relaxation_parameters': s_q,
            'equilibrium': [qy, qy*qx/h, qy**2/h + 0.5*g*h**2, 0.],
        },
    ],
    'parameters': {LA: la, g: 1.},
}

s = pyLBM.Scheme(d)
print(s)

```

Three elementary schemes have been built, these three schemes are identical except for the equilibrium values of the non conserved moment and of the relaxation parameter: The first one is used to simulate the equation on h and the others to simulate the equation on q_x and q_y . For each scheme, the equilibrium value of the non conserved moment is equal to the flux of the corresponding equation: the equilibrium value of the k th scheme can so depend on all the conserved moments (and not only on those of the k th scheme).

The Boundary Conditions

The simulations are performed in a bounded domain with optional obstacles. Boundary conditions have then to be imposed on all the bounds. With pyLBM, the user can use the classical boundary conditions (classical for the lattice Boltzmann method) that are already implemented or implement his own conditions.

Note that periodical boundary conditions are used as default conditions. The corresponding label is -1 .

For a lattice Boltzmann method, we have to impose the incoming distribution functions on nodes outside the domain. We describe

- first, how the bounce back, the anti bounce back, and the Neumann conditions can be used,
- second, how personal boundary conditions can be implemented.

The classical conditions

The bounce back and anti bounce back conditions

The bounce back condition (*resp.* anti bounce back) is used to impose the odd moments (*resp.* even moments) on the bounds.

The Neumann conditions

How to implement new conditions

The storage

When you use pyLBM, a generated code is performed using the description of the scheme(s) (the velocities, the polynomials, the conserved moments, the equilibriums, ...). There are several generators already implemented

- NumPy
- Cython
- Pythran (work in progress)
- Loo.py (work in progress)

To have best performance following the generator, you need a specific storage of the moments and distribution functions arrays. For example, it is preferable to have a storage like $[n_v, n_x, n_y, n_z]$ in NumPy n_v is the number of velocities and n_x, n_y and n_z the grid size. It is due to the vectorized form of the algorithm. Whereas for Cython, it is preferable to have the storage $[n_x, n_y, n_z, n_v]$ using the pull algorithm.

So, we have implemented a storage class that always gives to the user the same access to the moments and distribution functions arrays but with a different storage in memory for the generator. This class is called Array.

It is really simple to create an array. You just need to give

- the number of velocities,
- the global grid size,
- the size of the fictitious point in each direction,
- the order of $[n_v, n_x, n_y, n_z]$ with the following indices
 - 0: n_v
 - 1: n_x
 - 2: n_y
 - 3: n_z

The default order is $[n_v, n_x, n_y, n_z]$.

- the mpi topology (optional)
- the type of the data (optional)

The default is double

2D example

Suppose that you want to create an array with a grid size $[5, 10]$ and 9 velocities with 1 cell in each direction for the fictitious domain.

```
In [25]: from pyLBM.storage import Array
import numpy as np
a = Array(9, [5, 10], [1, 1])

In [28]: for i in range(a.nv):
a[i] = i

In [29]: print(a[:])
```

```
[[[ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
  [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
  [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
  [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
  [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]]

[[ 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]
 [ 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]
 [ 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]
 [ 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]
 [ 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]]

[[ 2.  2.  2.  2.  2.  2.  2.  2.  2.  2.]
 [ 2.  2.  2.  2.  2.  2.  2.  2.  2.  2.]
 [ 2.  2.  2.  2.  2.  2.  2.  2.  2.  2.]
 [ 2.  2.  2.  2.  2.  2.  2.  2.  2.  2.]
 [ 2.  2.  2.  2.  2.  2.  2.  2.  2.  2.]]

[[ 3.  3.  3.  3.  3.  3.  3.  3.  3.  3.]
 [ 3.  3.  3.  3.  3.  3.  3.  3.  3.  3.]
 [ 3.  3.  3.  3.  3.  3.  3.  3.  3.  3.]
 [ 3.  3.  3.  3.  3.  3.  3.  3.  3.  3.]
 [ 3.  3.  3.  3.  3.  3.  3.  3.  3.  3.]]

[[ 4.  4.  4.  4.  4.  4.  4.  4.  4.  4.]
 [ 4.  4.  4.  4.  4.  4.  4.  4.  4.  4.]
 [ 4.  4.  4.  4.  4.  4.  4.  4.  4.  4.]
 [ 4.  4.  4.  4.  4.  4.  4.  4.  4.  4.]
 [ 4.  4.  4.  4.  4.  4.  4.  4.  4.  4.]]

[[ 5.  5.  5.  5.  5.  5.  5.  5.  5.  5.]
 [ 5.  5.  5.  5.  5.  5.  5.  5.  5.  5.]
 [ 5.  5.  5.  5.  5.  5.  5.  5.  5.  5.]
 [ 5.  5.  5.  5.  5.  5.  5.  5.  5.  5.]
 [ 5.  5.  5.  5.  5.  5.  5.  5.  5.  5.]]

[[ 6.  6.  6.  6.  6.  6.  6.  6.  6.  6.]
 [ 6.  6.  6.  6.  6.  6.  6.  6.  6.  6.]
 [ 6.  6.  6.  6.  6.  6.  6.  6.  6.  6.]
 [ 6.  6.  6.  6.  6.  6.  6.  6.  6.  6.]
 [ 6.  6.  6.  6.  6.  6.  6.  6.  6.  6.]]

[[ 7.  7.  7.  7.  7.  7.  7.  7.  7.  7.]
 [ 7.  7.  7.  7.  7.  7.  7.  7.  7.  7.]
 [ 7.  7.  7.  7.  7.  7.  7.  7.  7.  7.]
 [ 7.  7.  7.  7.  7.  7.  7.  7.  7.  7.]
```

```
[ 7.  7.  7.  7.  7.  7.  7.  7.  7.  7.]

[[ 8.  8.  8.  8.  8.  8.  8.  8.  8.  8.]
 [ 8.  8.  8.  8.  8.  8.  8.  8.  8.  8.]
 [ 8.  8.  8.  8.  8.  8.  8.  8.  8.  8.]
 [ 8.  8.  8.  8.  8.  8.  8.  8.  8.  8.]
 [ 8.  8.  8.  8.  8.  8.  8.  8.  8.  8.]]

In [30]: b = Array(9, [5, 10], [1, 1], sorder=[2, 1, 0])
        for i in range(b.nv):
            b[i] = i

In [31]: print(b[:])

[[ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]]

[[ 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]
 [ 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]
 [ 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]
 [ 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]
 [ 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]]

[[ 2.  2.  2.  2.  2.  2.  2.  2.  2.  2.]
 [ 2.  2.  2.  2.  2.  2.  2.  2.  2.  2.]
 [ 2.  2.  2.  2.  2.  2.  2.  2.  2.  2.]
 [ 2.  2.  2.  2.  2.  2.  2.  2.  2.  2.]
 [ 2.  2.  2.  2.  2.  2.  2.  2.  2.  2.]]

[[ 3.  3.  3.  3.  3.  3.  3.  3.  3.  3.]
 [ 3.  3.  3.  3.  3.  3.  3.  3.  3.  3.]
 [ 3.  3.  3.  3.  3.  3.  3.  3.  3.  3.]
 [ 3.  3.  3.  3.  3.  3.  3.  3.  3.  3.]
 [ 3.  3.  3.  3.  3.  3.  3.  3.  3.  3.]]

[[ 4.  4.  4.  4.  4.  4.  4.  4.  4.  4.]
 [ 4.  4.  4.  4.  4.  4.  4.  4.  4.  4.]
 [ 4.  4.  4.  4.  4.  4.  4.  4.  4.  4.]
 [ 4.  4.  4.  4.  4.  4.  4.  4.  4.  4.]
 [ 4.  4.  4.  4.  4.  4.  4.  4.  4.  4.]]

[[ 5.  5.  5.  5.  5.  5.  5.  5.  5.  5.]
 [ 5.  5.  5.  5.  5.  5.  5.  5.  5.  5.]
 [ 5.  5.  5.  5.  5.  5.  5.  5.  5.  5.]
 [ 5.  5.  5.  5.  5.  5.  5.  5.  5.  5.]
 [ 5.  5.  5.  5.  5.  5.  5.  5.  5.  5.]]

[[ 6.  6.  6.  6.  6.  6.  6.  6.  6.  6.]
 [ 6.  6.  6.  6.  6.  6.  6.  6.  6.  6.]
 [ 6.  6.  6.  6.  6.  6.  6.  6.  6.  6.]
 [ 6.  6.  6.  6.  6.  6.  6.  6.  6.  6.]
 [ 6.  6.  6.  6.  6.  6.  6.  6.  6.  6.]]

[[ 7.  7.  7.  7.  7.  7.  7.  7.  7.  7.]
 [ 7.  7.  7.  7.  7.  7.  7.  7.  7.  7.]
 [ 7.  7.  7.  7.  7.  7.  7.  7.  7.  7.]
 [ 7.  7.  7.  7.  7.  7.  7.  7.  7.  7.]
```

```
[ 7.  7.  7.  7.  7.  7.  7.  7.  7.  7.]

[[ 8.  8.  8.  8.  8.  8.  8.  8.  8.  8.]
 [ 8.  8.  8.  8.  8.  8.  8.  8.  8.  8.]
 [ 8.  8.  8.  8.  8.  8.  8.  8.  8.  8.]
 [ 8.  8.  8.  8.  8.  8.  8.  8.  8.  8.]
 [ 8.  8.  8.  8.  8.  8.  8.  8.  8.  8.]
```

You can see that the access of the data is the same for a et b whereas the sorder is not the same.

If we look at the `array` attribute which is the real storage of our data

```
In [32]: a.array.shape
```

```
Out[32]: (9, 5, 10)
```

```
In [33]: b.array.shape
```

```
Out[33]: (10, 5, 9)
```

you can see that it is not the same and it is exactly what we want. To do that, we use the `swapaxes` of numpy and we use this representation to have an access to our data.

Access to the data with the conserved moments

When you discribe your scheme, you define the conserved moments. It is usefull to have a direct acces to these moments by giving their name and not their indices in the array. So, it is possible to specify where are the conserved moments in the array.

Let define conserved moments using sympy symbol.

```
In [35]: import sympy
         rho, u, v = sympy.symbols("rho, u, v")
```

We indicate to pyLBM where are located these conserved moments in our array by giving a list of two elements: the first one is the scheme number and the second one the index in this scheme.

```
In [45]: a.set_conserved_moments({rho: [0, 0], u: [0, 2], v: [0, 1]}, [0, 9])
```

```
In [46]: a[rho]
```

```
Out[46]: array([[ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
 [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
 [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
 [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
 [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.]])
```

```
In [47]: a[u]
```

```
Out[47]: array([[ 2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.],
 [ 2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.],
 [ 2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.],
 [ 2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.],
 [ 2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.,  2.]])
```

```
In [48]: a[v]
```

```
Out[48]: array([[ 1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.],
 [ 1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.],
 [ 1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.],
 [ 1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.],
 [ 1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.]])
```

```
In [ ]:
```

Tutorial

Transport in 1D

In this tutorial, we test the most simple lattice Boltzmann scheme D_1Q_2 on two classical hyperbolic scalar equations: the advection equation and the Burger's equation.

The advection equation

The problem reads

$$\partial_t u + c \partial_x u = 0, \quad t > 0, \quad x \in (0, 1),$$

where c is a constant scalar (typically $c = 1$). Additional boundary and initial conditions will be given in the following.

The numerical simulation of this equation by a lattice Boltzmann scheme consists in the approximation of the solution on discret points of $(0, 1)$ at discret instants.

The spatial mesh is defined by using a numpy array. To simplify, the mesh is supposed to be uniform.

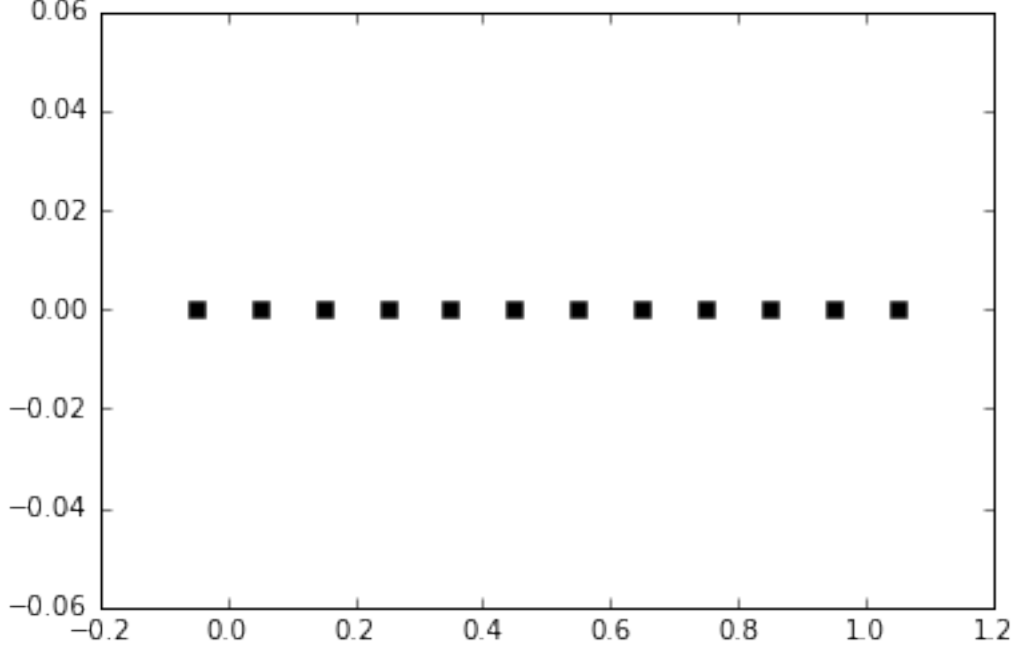
First, we import the package numpy and we create the spatial mesh. One phantom cell has to be added at each edge of the domain for the treatment of the boundary conditions.

```
In [1]: %matplotlib inline
```

```
In [2]: import numpy as np
import pylab as plt
```

```
def mesh(N):
    xmin, xmax = 0., 1.
    dx = 1./N
    x = np.linspace(xmin-.5*dx, xmax+.5*dx, N+2)
    return x
```

```
x = mesh(10)
plt.plot(x, 0.*x, 'sk')
plt.show()
```

To simulate this equation, we use the D_1Q_2 scheme given by

- two velocities $v_0 = -1$, $v_1 = 1$, with associated distribution functions f_0 and f_1 ,
- a space step Δx and a time step Δt , the ration $\lambda = \Delta x / \Delta t$ is called the scheme velocity,
- two moments $m_0 = \sum_{i=0}^1 f_i$ and $m_1 = \lambda \sum_{i=0}^1 v_i f_i$ and their equilibrium values $m_0^e = m_0$, $m_1^e = c m_0$,
- a relaxation parameter s lying in $[0, 2]$.

In order to prepare the formalism of the package pyLBM, we introduce the two polynomials that define the moments: $P_0 = 1$ and $P_1 = \lambda X$, such that

$$m_k = \sum_{i=0}^1 P_k(v_i) f_i.$$

The transformation $(f_0, f_1) \mapsto (m_0, m_1)$ is invertible if, and only if, the polynomials (P_0, P_1) is a free set over the stencil of velocities.

The lattice Boltzmann method consists to compute the distribution functions f_0 and f_1 in each point of the lattice x and at each time $t^n = n\Delta t$. A step of the scheme can be read as a splitting between the relaxation phase and the transport phase:

- relaxation:

$$m_1^*(t, x) = (1 - s) m_1(t, x) + s m_1^e(t, x).$$

- m2f:

$$\begin{aligned} f_0^*(t, x) &= (m_0(t, x) - m_1^*(t, x)/\lambda)/2, \\ f_1^*(t, x) &= (m_0(t, x) + m_1^*(t, x)/\lambda)/2. \end{aligned}$$

- transport:

$$f_0(t + \Delta t, x) = f_0^*(t, x + \Delta x), \quad f_1(t + \Delta t, x) = f_1^*(t, x - \Delta x).$$

- f2m:

$$\begin{aligned}m_0(t + \Delta t, x) &= f_0(t + \Delta t, x) + f_1(t + \Delta t, x), \\m_1(t + \Delta t, x) &= -\lambda f_0(t + \Delta t, x) + \lambda f_1(t + \Delta t, x).\end{aligned}$$

The moment of order 0, m_0 , being the only one conserved during the relaxation phase, the equivalent equation of this scheme reads at first order

$$\partial_t m_0 + \partial_x m_1^e = \mathcal{O}(\Delta t).$$

We implement a function equilibrium that computes the equilibrium value m_1^e , the moment of order 0, m_0 , and the velocity c being given in argument.

```
In [3]: def equilibrium(m0, c):  
        return c*m0
```

Then, we create two vectors m_0 and m_1 with shape the shape of the mesh and initialize them. The moment of order 0 should contain the initial value of the unknown u and the moment of order 1 the corresponding equilibrium value.

We create also two vectors f_0 and f_1 .

```
In [4]: def initialize(mesh, c, la):  
        m0 = np.zeros(mesh.shape)  
        m0[np.logical_and(mesh<0.5, mesh>0.25)] = 1.  
        m1 = equilibrium(m0, c)  
        f0, f1 = np.empty(m0.shape), np.empty(m0.shape)  
        m2f(f0, f1, m0, m1, la)  
        return f0, f1, m0, m1
```

And finally, we implement the four elementary functions f2m, relaxation, m2f, and transport. In the transport function, the boundary conditions should be implemented: we will use periodic conditions by copying the informations in the phantom cells.

```
In [5]: def f2m(f0, f1, m0, m1, la):  
        m0[:] = f0 + f1  
        m1[:] = la*(f1 - f0)  
  
        def m2f(f0, f1, m0, m1, la):  
            f0[:] = 0.5*(m0-m1/la)  
            f1[:] = 0.5*(m0+m1/la)  
  
        def relaxation(m0, m1, c, s):  
            m1[:] = (1-s)*m1 + s*equilibrium(m0, c)  
  
        def transport(f0, f1):  
            #periodical boundary conditions  
            f0[-1] = f0[1]  
            f1[0] = f1[-2]  
            #transport  
            f0[1:-1] = f0[2:]  
            f1[1:-1] = f1[:-2]
```

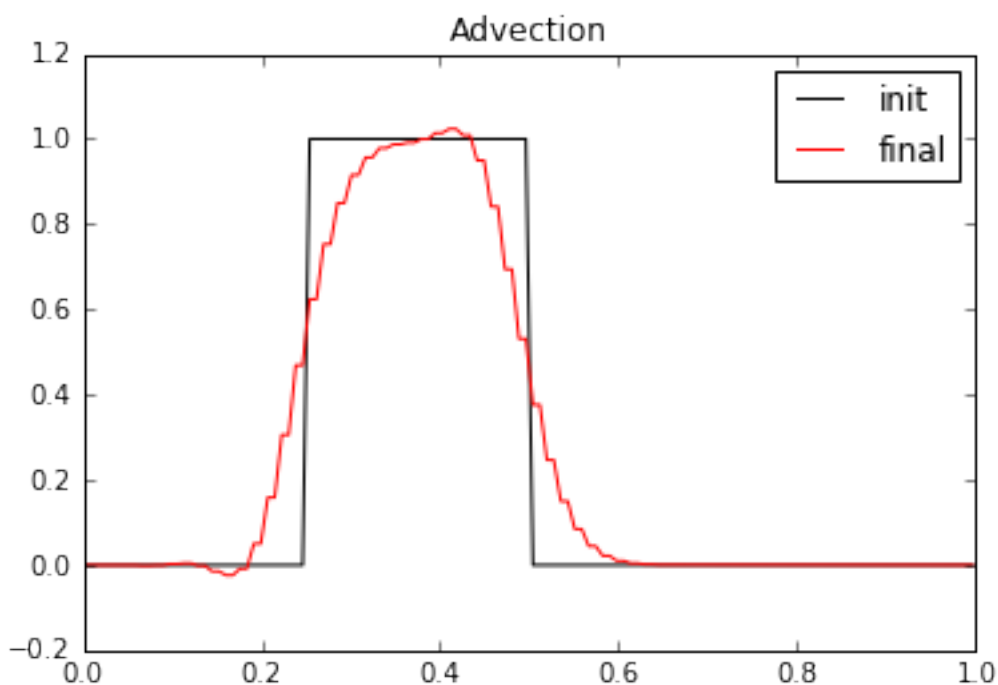
We compute and we plot the numerical solution at time $T_f = 2$.

```
In [6]: # parameters  
        c = .5 # velocity for the transport equation  
        Tf = 2. # final time  
        N = 128 # number of points in space  
        la = 1. # scheme velocity
```

```

s = 1.8 # relaxation parameter
# initialization
x = mesh(N)
f0, f1, m0, m1 = initialize(x, c, la)
t = 0
dt = (x[1]-x[0])/la
plt.figure(1)
plt.clf()
plt.plot(x[1:-1], m0[1:-1], 'k', label='init')
while t<Tf:
    t += dt
    relaxation(m0, m1, c, s)
    m2f(f0, f1, m0, m1, la)
    transport(f0, f1)
    f2m(f0, f1, m0, m1, la)
plt.plot(x[1:-1], m0[1:-1], 'r', label='final')
plt.legend()
plt.title('Advection')
plt.show()

```



The Burger's equation

The problem reads

$$\partial_t u + \frac{1}{2} \partial_x u^2 = 0, \quad t > 0, \quad x \in (0, 1).$$

The previous D_1Q_2 scheme can simulate the Burger's equation by modifying the equilibrium value of the moment of order 1 m_1^e . It now reads $m_1^e = m_0^2/2$.

More generally, the simulated equation is into the conservative form

$$\partial_t u + \partial_x \varphi(u) = 0, \quad t > 0, \quad x \in (0, 1),$$

the equilibrium has to be taken to $m_1^e = \varphi(m_0)$.

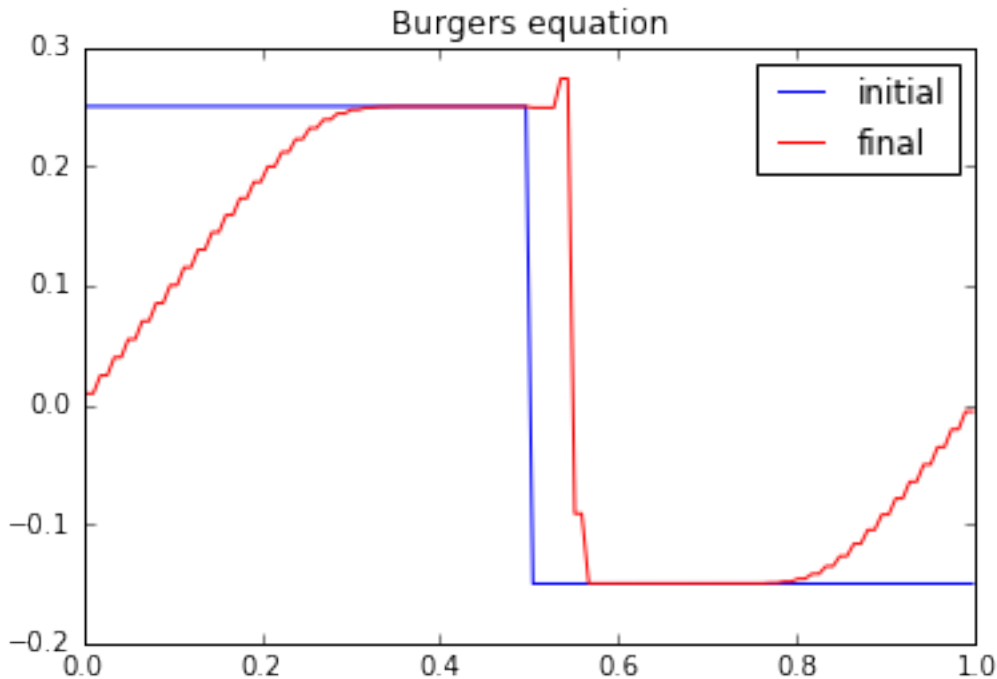
We just have to modify the equilibrium and the initialization of the previous example to simulate the Burger's equation. The initial condition can be a discontinuous function in order to simulate Riemann problems. Note that the function `f2m`, `m2f`, `relaxation`, and `transport` are unchanged.

```
In [7]: def equilibrium(m0):
        return .5*m0**2

def initialize(mesh, la):
    ug, ud = 0.25, -0.15
    xmin, xmax = .5*np.sum(mesh[:2]), .5*np.sum(mesh[-2:])
    xc = xmin + .5*(xmax-xmin)
    m0 = ug*(mesh<xc) + ud*(mesh>xc) + .5*(ug+ud)*(mesh==xc)
    m1 = equilibrium(m0)
    f0 = np.empty(m0.shape)
    f1 = np.empty(m0.shape)
    return f0, f1, m0, m1

def relaxation(m0, m1, s):
    m1[:] = (1-s)*m1 + s*equilibrium(m0)

# parameters
Tf = 1. # final time
N = 128 # number of points in space
la = 1. # scheme velocity
s = 1.8 # relaxation parameter
# initialization
x = mesh(N) # mesh
dx = x[1]-x[0] # space step
dt = dx/la # time step
f0, f1, m0, m1 = initialize(x, la)
plt.figure(1)
plt.plot(x[1:-1], m0[1:-1], 'b', label='initial')
# time loops
t = 0.
while (t<Tf):
    t += dt
    relaxation(m0, m1, s)
    m2f(f0, f1, m0, m1, la)
    transport(f0, f1)
    f2m(f0, f1, m0, m1, la)
plt.plot(x[1:-1], m0[1:-1], 'r', label='final')
plt.title('Burgers equation')
plt.legend(loc='best')
plt.show()
```



We can test different values of the relaxation parameter s . In particular, we observe that the scheme remains stable if $s \in [0, 2]$. More s is small, more the numerical diffusion is important and if s is close to 2, oscillations appear behind the shock.

In order to simulate a Riemann problem, the boundary conditions have to be modified. A classical way is to impose entry conditions for hyperbolic problems. The lattice Boltzmann methods lend themselves very well to that conditions: the scheme only needs the distributions corresponding to a velocity that goes inside the domain. Nevertheless, on a physical edge where the flux is going outside, a non physical distribution that goes inside has to be imposed. A first simple way is to leave the initial value: this is correct while the discontinuity does not reach the edge. A second way is to impose Neumann condition by repeating the inner value.

We modify the previous script to take into account these new boundary conditions.

```
In [8]: def transport(f0, f1):
        # Neumann boundary conditions
        f0[-1] = f0[-2]
        f1[0] = f1[1]
        # transport
        f0[1:-1] = f0[2:]
        f1[1:-1] = f1[:-2]

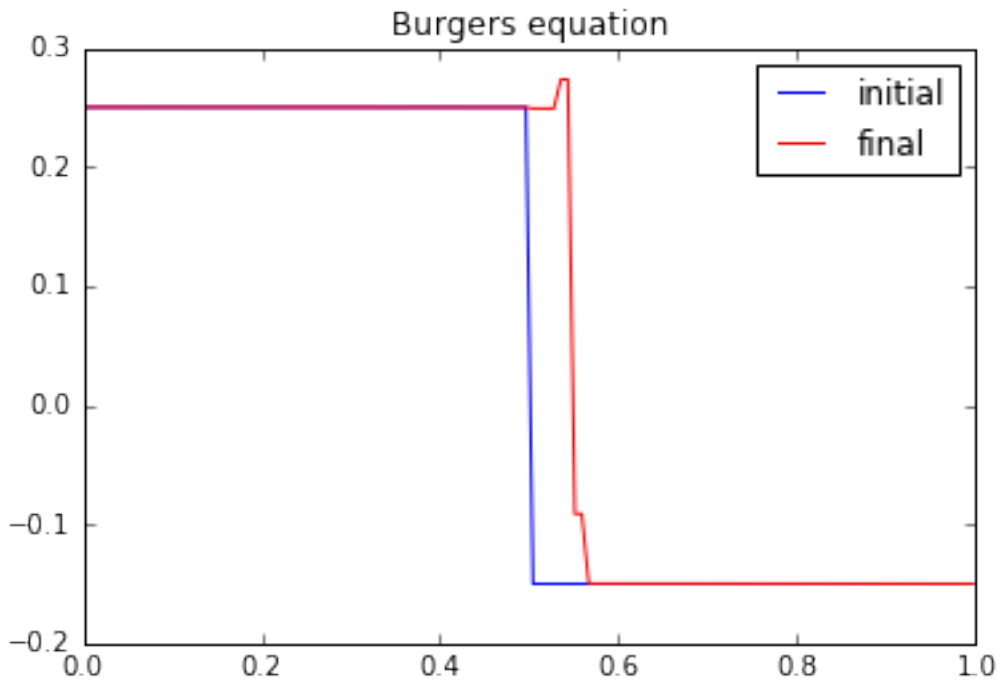
        # parameters
        Tf = 1. # final time
        N = 128 # number of points in space
        la = 1. # scheme velocity
        s = 1.8 # relaxation parameter

        # initialization
        x = mesh(N) # mesh
        dx = x[1]-x[0] # space step
        dt = dx/la # time step
        f0, f1, m0, m1 = initialize(x, la)
        plt.figure(1)
        plt.plot(x[1:-1], m0[1:-1], 'b', label='initial')
```

```

# time loops
t = 0.
while (t<Tf):
    t += dt
    relaxation(m0, m1, s)
    m2f(f0, f1, m0, m1, la)
    transport(f0, f1)
    f2m(f0, f1, m0, m1, la)
plt.plot(x[1:-1], m0[1:-1], 'r', label='final')
plt.title('Burgers equation')
plt.legend(loc='best')
plt.show()

```



In []:

The wave equation in 1D

In this tutorial, we test a very classical lattice Boltzmann scheme D_1Q_3 on the wave equation.

The problem reads

$$\partial_{tt}\rho = c^2\partial_{xx}\rho, \quad t > 0, \quad x \in (0, 2\pi),$$

where c is a constant scalar. In this session, two different kinds of boundary conditions will be considered:

- periodic conditions $\rho(0) = \rho(2\pi)$,
- Homogeneous Dirichlet conditions $\rho(0) = \rho(2\pi) = 0$.

The problem is transformed into a one order system:

$$\begin{aligned} \partial_t \rho + \partial_x q &= 0, & t > 0, & \quad x \in (0, 2\pi), \\ \partial_t q + c^2 \partial_x \rho &= 0, & t > 0, & \quad x \in (0, 2\pi). \end{aligned}$$

The scheme D_1Q_3

The numerical simulation of this equation by a lattice Boltzmann scheme consists in the approximation of the solution on discret points of $(0, 2\pi)$ at discret instants.

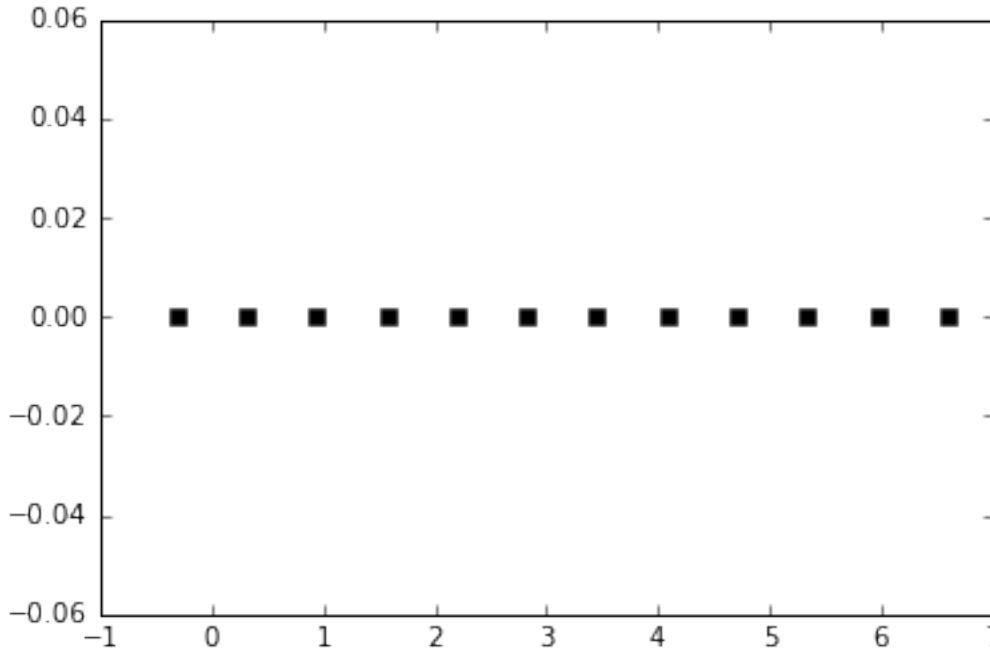
The spatial mesh is defined by using a numpy array. To simplify, the mesh is supposed to be uniform.

First, we import the package numpy and we create the spatial mesh. One phantom cell has to be added at each bound for the treatment of the boundary conditions.

```
In [1]: %matplotlib inline
In [2]: import numpy as np
import pylab as plt

def mesh(N):
    xmin, xmax = 0., 2.*np.pi
    dx = (xmax-xmin)/N
    x = np.linspace(xmin-.5*dx, xmax+.5*dx, N+2)
    return x

x = mesh(10)
plt.plot(x, 0.*x, 'sk')
plt.show()
```



To simulate this system of equations, we use the D_1Q_3 scheme given by

- three velocities $v_0 = 0$, $v_1 = 1$, and $v_2 = -1$, with associated distribution functions f_0 , f_1 , and f_2 ,
- a space step Δx and a time step Δt , the ration $\lambda = \Delta x / \Delta t$ is called the scheme velocity,
- three moments

$$m_0 = \sum_{i=0}^2 f_i, \quad m_1 = \lambda \sum_{i=0}^2 v_i f_i, \quad m_2 = \frac{\lambda^2}{2} \sum_{i=0}^2 v_i^2 f_i,$$

and their equilibrium values $m_0^e = m_0$, $m_1^e = m_1$, and $m_2^e = c^2/2 m_0$.

- a relaxation parameter s lying in $[0, 2]$.

In order to prepare the formalism of the package pyLBM, we introduce the three polynomials that define the moments: $P_0 = 1$, $P_1 = \lambda X$, and $P_2 = \lambda^2/2X^2$, such that

$$m_k = \sum_{i=0}^2 P_k(v_i) f_i.$$

The transformation $(f_0, f_1, f_2) \mapsto (m_0, m_1, m_2)$ is invertible if, and only if, the polynomials (P_0, P_1, P_2) is a free set over the stencil of velocities.

The lattice Boltzmann method consists to compute the distribution functions f_0 , f_1 , and f_2 in each point of the lattice x and at each time $t^n = n\Delta t$. A step of the scheme can be read as a splitting between the relaxation phase and the transport phase:

- relaxation:

$$m_2^*(t, x) = (1 - s) m_2(t, x) + s m_2^e(t, x).$$

- m2f:

$$\begin{aligned} f_0^*(t, x) &= m_0(t, x) - 2 m_2^*(t, x)/\lambda^2, \\ f_1^*(t, x) &= m_1(t, x)/(2\lambda) + m_2^*(t, x)/\lambda^2, \\ f_2^*(t, x) &= -m_1(t, x)/(2\lambda) + m_2^*(t, x)/\lambda^2. \end{aligned}$$

- transport:

$$\begin{aligned} f_0(t + \Delta t, x) &= f_0^*(t, x), \\ f_1(t + \Delta t, x) &= f_1^*(t, x - \Delta x), \\ f_2(t + \Delta t, x) &= f_2^*(t, x + \Delta x). \end{aligned}$$

- f2m:

$$\begin{aligned} m_0(t + \Delta t, x) &= f_0(t + \Delta t, x) + f_1(t + \Delta t, x) + f_2(t + \Delta t, x), \\ m_1(t + \Delta t, x) &= \lambda f_1(t + \Delta t, x) - \lambda f_2(t + \Delta t, x), \\ m_2(t + \Delta t, x) &= \frac{1}{2}\lambda^2 f_1(t + \Delta t, x) + \frac{1}{2}\lambda^2 f_2(t + \Delta t, x). \end{aligned}$$

The moments of order 0, m_0 , and of order 1, m_1 , being conserved during the relaxation phase, the equivalent equations of this scheme read at first order

$$\begin{aligned} \partial_t m_0 + \partial_x m_1 &= \mathcal{O}(\Delta t), \\ \partial_t m_1 + 2\partial_x m_2^e &= \mathcal{O}(\Delta t). \end{aligned}$$

We implement a function equilibrium that computes the equilibrium value m_2^e , the moment of order 0, m_0 , and the velocity c being given in argument.

```
In [3]: def equilibrium(m0, c):  
        return .5*c**2*m0
```

We create three vectors m_0 , m_1 , and m_2 with shape the shape of the mesh and initialize them. The moments of order 0 and 1 should contain the initial value of the unknowns ρ and q , and the moment of order 2 the corresponding equilibrium value.

We create also three vectors f_0 , f_1 and f_2 .


```
In [4]: def initialize(mesh, c, la):
        m0 = np.sin(mesh)
        m1 = np.zeros(mesh.shape)
        m2 = equilibrium(m0, c)
        f0 = np.empty(m0.shape)
        f1 = np.empty(m0.shape)
        f2 = np.empty(m0.shape)
        return f0, f1, f2, m0, m1, m2
```

Periodic boundary conditions

We implement the four elementary functions `f2m`, `relaxation`, `m2f`, and `transport`. In the `transport` function, the boundary conditions should be implemented: we will use periodic conditions by copying the informations in the phantom cells.

```
In [5]: def f2m(f0, f1, f2, m0, m1, m2, la):
        m0[:] = f0 + f1 + f2
        m1[:] = la * (f2 - f1)
        m2[:] = .5* la**2 * (f1 + f2)

        def m2f(f0, f1, f2, m0, m1, m2, la):
            f0[:] = m0 - 2./la**2 * m2
            f1[:] = -.5/la * m1 + 1/la**2 * m2
            f2[:] = .5/la * m1 + 1/la**2 * m2

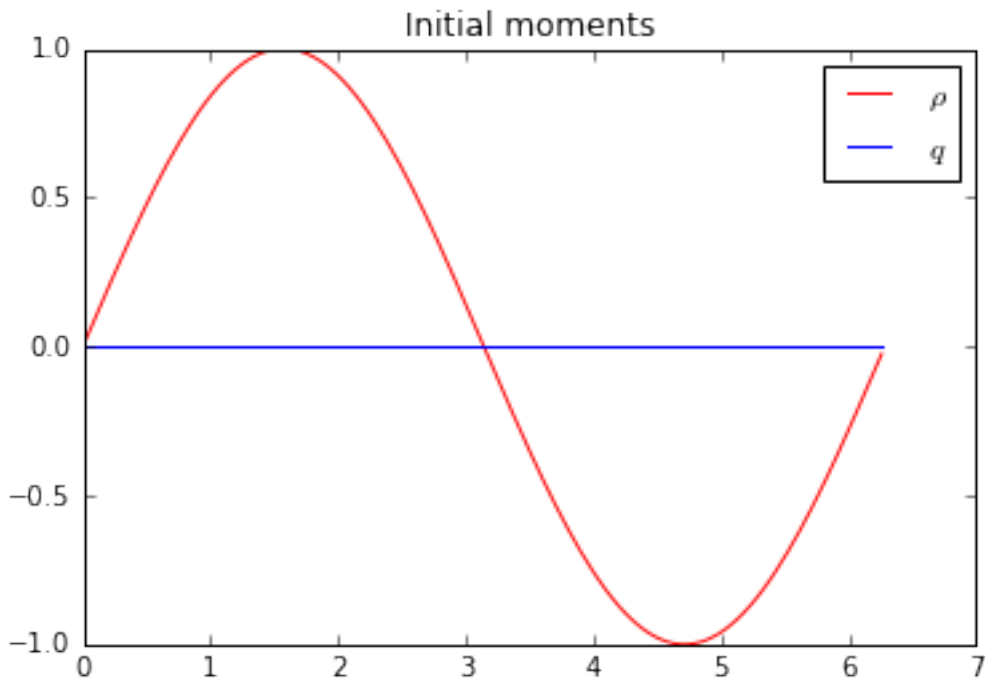
        def relaxation(m0, m1, m2, c, s):
            m2[:] *= (1-s)
            m2[:] += s*equilibrium(m0, c)

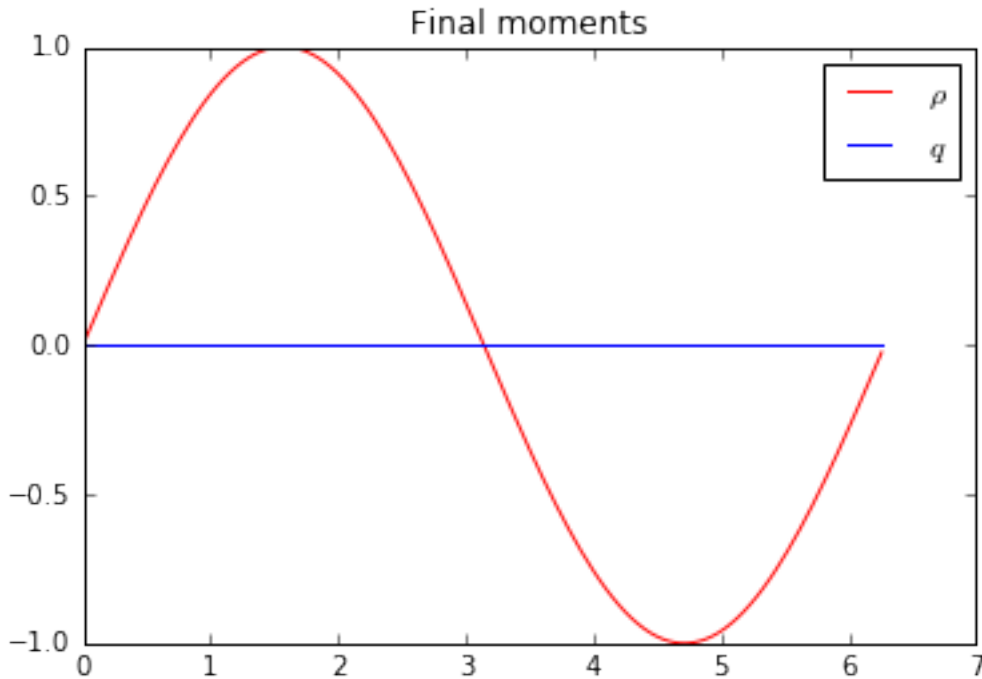
        def transport(f0, f1, f2):
            # periodic boundary conditions
            f1[-1] = f1[1]
            f2[0] = f2[-2]
            # transport
            f1[1:-1] = f1[2:]
            f2[1:-1] = f2[:-2]
```

We compute and we plot the numerical solution at time $T_f = 2\pi$.

```
In [6]: # parameters
        c = 1 # velocity for the transport equation
        Tf = 2.*np.pi # final time
        N = 128 # number of points in space
        la = 1. # scheme velocity
        s = 2. # relaxation parameter
        # initialization
        x = mesh(N) # mesh
        dx = x[1]-x[0] # space step
        dt = dx/la # time step
        f0, f1, f2, m0, m1, m2 = initialize(x, c, la)
        plt.figure(1)
        plt.plot(x[1:-1], m0[1:-1], 'r', label=r'$\rho$')
        plt.plot(x[1:-1], m1[1:-1], 'b', label=r'$q$')
        plt.title('Initial moments')
        plt.legend(loc='best')
        # time loops
        nt = int(Tf/dt)
        m2f(f0, f1, f2, m0, m1, m2, la)
```

```
for k in range(nt):
    transport(f0, f1, f2)
    f2m(f0, f1, f2, m0, m1, m2, la)
    relaxation(m0, m1, m2, c, s)
    m2f(f0, f1, f2, m0, m1, m2, la)
plt.figure(2)
plt.plot(x[1:-1], m0[1:-1], 'r', label=r'$\rho$')
plt.plot(x[1:-1], m1[1:-1], 'b', label=r'$q$')
plt.title('Final moments')
plt.legend(loc='best')
plt.show()
```





Anti bounce back conditions

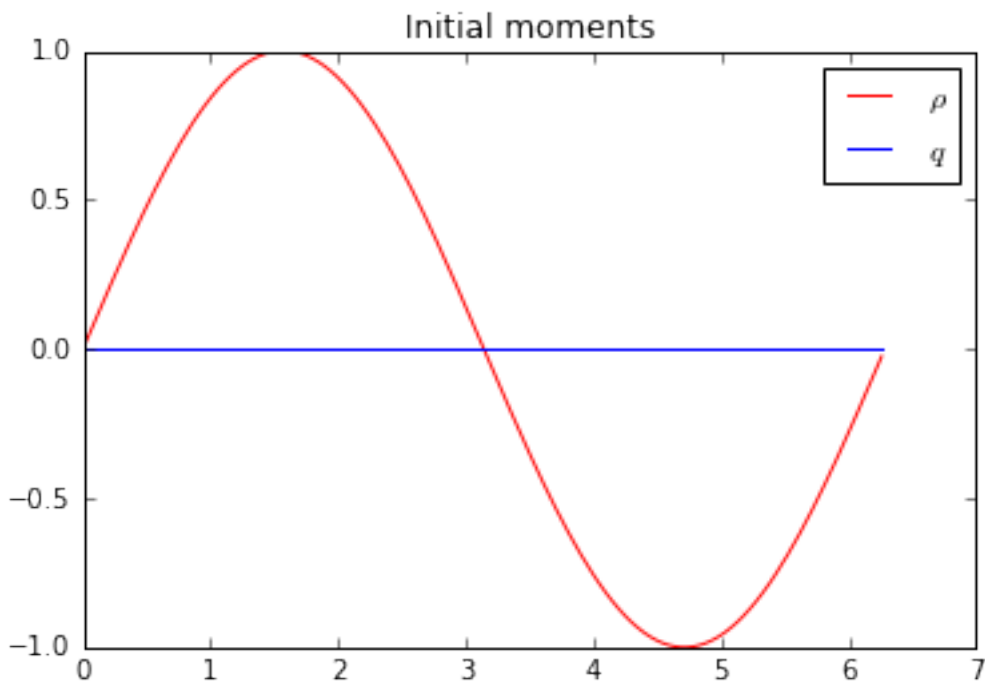
In order to take into account homogenous Dirichlet conditions over ρ , we introduce the bounce back conditions. At edge $x = 0$, two points are involved: $x_0 = -\Delta x/2$ and $x_1 = \Delta x/2$. We impose $f_1(x_0) = -f_2(x_1)$. And at edge $x = 2\pi$, the two involved points are x_N and x_{N+1} . We impose $f_2(x_{N+1}) = -f_1(x_N)$.

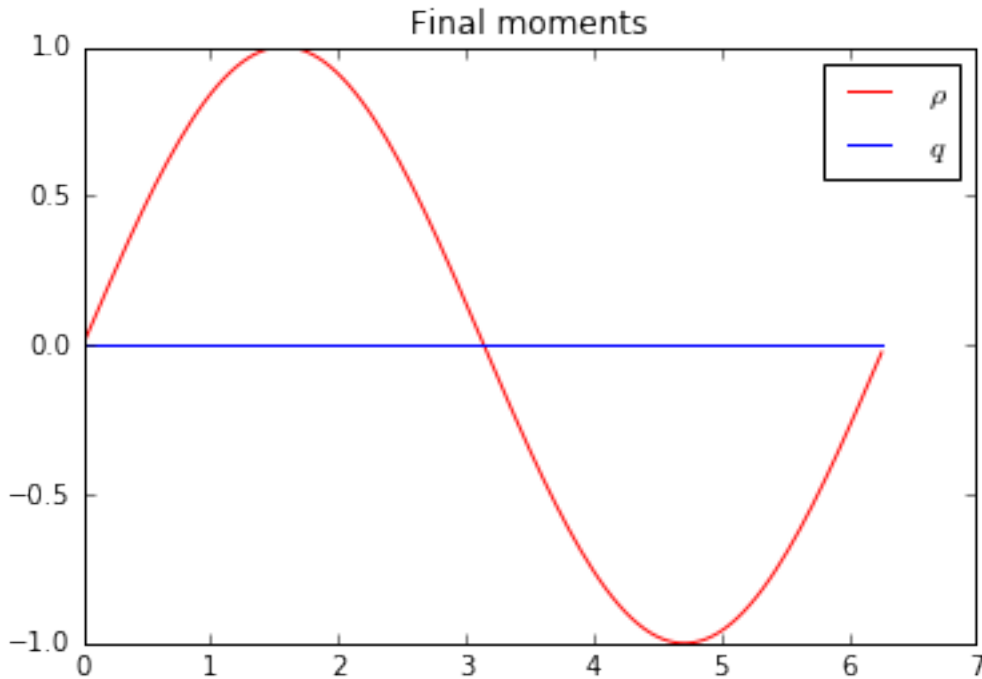
We modify the transport function to impose anti bounce back conditions. We can compare the solutions obtained with the two different boundary conditions.

```
In [7]: def transport(f0, f1, f2):
        # anti bounce back boundary conditions
        f1[-1] = -f2[-2]
        f2[0] = -f1[1]
        # transport
        f1[1:-1] = f1[2:]
        f2[1:-1] = f2[:2]

        # parameters
        c = 1 # velocity for the transport equation
        Tf = 2*np.pi # final time
        N = 128 # number of points in space
        la = 1. # scheme velocity
        s = 2. # relaxation parameter
        # initialization
        x = mesh(N) # mesh
        dx = x[1]-x[0] # space step
        dt = dx/la # time step
        f0, f1, f2, m0, m1, m2 = initialize(x, c, la)
        plt.figure(1)
        plt.plot(x[1:-1], m0[1:-1], 'r', label=r'$\rho$')
        plt.plot(x[1:-1], m1[1:-1], 'b', label=r'$q$')
        plt.title('Initial moments')
```

```
plt.legend(loc='best')
# time loops
nt = int(Tf/dt)
m2f(f0, f1, f2, m0, m1, m2, la)
for k in range(nt):
    transport(f0, f1, f2)
    f2m(f0, f1, f2, m0, m1, m2, la)
    relaxation(m0, m1, m2, c, s)
    m2f(f0, f1, f2, m0, m1, m2, la)
plt.figure(2)
plt.plot(x[1:-1], m0[1:-1], 'r', label=r'$\rho$')
plt.plot(x[1:-1], m1[1:-1], 'b', label=r'$q$')
plt.title('Final moments')
plt.legend(loc='best')
plt.show()
```





In []:

The heat equation in 1D

In this tutorial, we test a very classical lattice Boltzmann scheme D_1Q_3 on the heat equation.

The problem reads

$$\begin{aligned} \partial_t u &= \mu \partial_{xx} u, \quad t > 0, \quad x \in (0, 1), \\ u(0) &= u(1) = 0, \\ \text{where : } \mu &: \text{'}\mu\text{' is a constant scalar.} \end{aligned}$$

```
In [1]: from __future__ import print_function, division
        from six.moves import range
        %matplotlib inline
```

The scheme D_1Q_3

The numerical simulation of this equation by a lattice Boltzmann scheme consists in the approximation of the solution on discrete points of $(0, 1)$ at discrete instants.

To simulate this system of equations, we use the D_1Q_3 scheme given by

- three velocities $v_0 = 0$, $v_1 = 1$, and $v_2 = -1$, with associated distribution functions f_0 , f_1 , and f_2 ,
- a space step Δx and a time step Δt , the ratio $\lambda = \Delta x / \Delta t$ is called the scheme velocity,
- three moments

$$m_0 = \sum_{i=0}^2 f_i, \quad m_1 = \sum_{i=0}^2 v_i f_i, \quad m_2 = \frac{1}{2} \sum_{i=0}^2 v_i^2 f_i,$$

and their equilibrium values m_0^e , m_1^e , and m_2^e . * two relaxation parameters s_1 and s_2 lying in $[0, 2]$.

In order to use the formalism of the package pyLBM, we introduce the three polynomials that define the moments: $P_0 = 1$, $P_1 = X$, and $P_2 = X^2/2$, such that

$$m_k = \sum_{i=0}^2 P_k(v_i) f_i.$$

The transformation $(f_0, f_1, f_2) \mapsto (m_0, m_1, m_2)$ is invertible if, and only if, the polynomials (P_0, P_1, P_2) is a free set over the stencil of velocities.

The lattice Boltzmann method consists to compute the distribution functions f_0 , f_1 , and f_2 in each point of the lattice x and at each time $t^n = n\Delta t$. A step of the scheme can be read as a splitting between the relaxation phase and the transport phase:

- relaxation:

$$\begin{aligned} m_1^*(t, x) &= (1 - s_1) m_1(t, x) + s_1 m_1^e(t, x), \\ m_2^*(t, x) &= (1 - s_2) m_2(t, x) + s_2 m_2^e(t, x). \end{aligned}$$

- m2f:

$$\begin{aligned} f_0^*(t, x) &= m_0(t, x) - 2m_2^*(t, x), \\ f_1^*(t, x) &= m_1^*(t, x)/2 + m_2^*(t, x), \\ f_2^*(t, x) &= -m_1^*(t, x)/2 + m_2^*(t, x). \end{aligned}$$

- transport:

$$\begin{aligned} f_0(t + \Delta t, x) &= f_0^*(t, x), \\ f_1(t + \Delta t, x) &= f_1^*(t, x - \Delta x), \\ f_2(t + \Delta t, x) &= f_2^*(t, x + \Delta x). \end{aligned}$$

- f2m:

$$\begin{aligned} m_0(t + \Delta t, x) &= f_0(t + \Delta t, x) + f_1(t + \Delta t, x) + f_2(t + \Delta t, x), \\ m_1(t + \Delta t, x) &= f_1(t + \Delta t, x) - f_2(t + \Delta t, x), \\ m_2(t + \Delta t, x) &= \frac{1}{2} f_1(t + \Delta t, x) + \frac{1}{2} f_2(t + \Delta t, x). \end{aligned}$$

The moment of order 0, m_0 , being conserved during the relaxation phase, a diffusive scaling $\Delta t = \Delta x^2$, yields to the following equivalent equation

$$\partial_t m_0 = 2\left(\frac{1}{s_1} - \frac{1}{2}\right) \partial_{xx} m_2^e + \mathcal{O}(\Delta x^2),$$

if $m_1^e = 0$. In order to be consistent with the heat equation, the following choice is done:

$$m_2^e = \frac{1}{2}u, \quad s_1 = \frac{2}{1 + 2\mu}, \quad s_2 = 1.$$

Using pyLBM

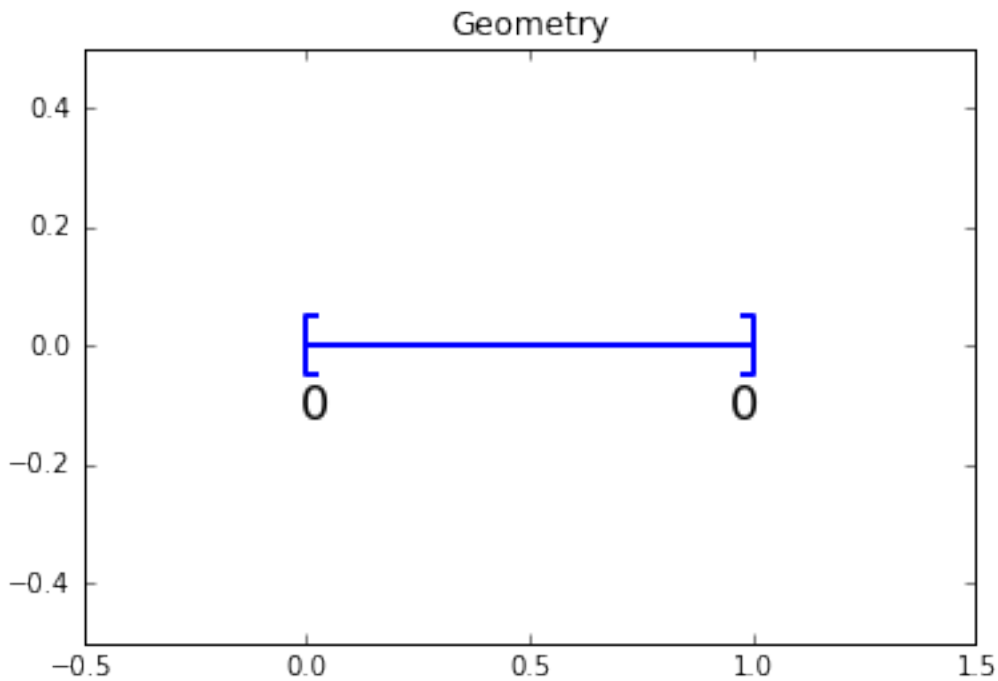
pyLBM uses Python dictionary to describe the simulation. In the following, we will build this dictionary step by step.

The geometry

In pyLBM, the geometry is defined by a box and a label for the boundaries.

```
In [2]: import pyLBM
import numpy as np
xmin, xmax = 0., 1.
dico_geom = {
    'box': {'x': [xmin, xmax], 'label': 0},
}
geom = pyLBM.Geometry(dico_geom)
print(geom)
geom.visualize(viewlabel=True)
```

```
Geometry informations
  spatial dimension: 1
  bounds of the box:
[[ 0.  1.]]
```



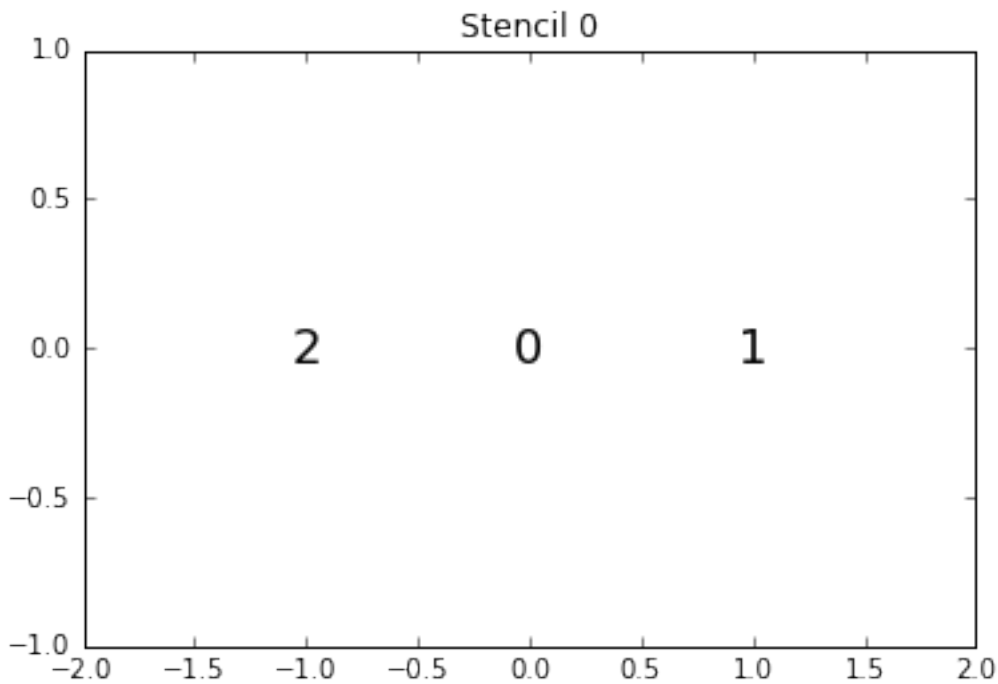
The stencil

pyLBM provides a class stencil that is used to define the discret velocities of the scheme. In this example, the stencil is composed by the velocities $v_0 = 0$, $v_1 = 1$ and $v_2 = -1$ numbered by $[0, 1, 2]$.

```
In [3]: dico_sten = {
    'dim': 1,
    'schemes': [{'velocities': list(range(3))}],
}
sten = pyLBM.Stencil(dico_sten)
print(sten)
sten.visualize()
```

Stencil informations

- * spatial dimension: 1
- * maximal velocity in each direction: [1]
- * minimal velocity in each direction: [-1]
- * Informations for each elementary stencil:
 - stencil 0
 - number of velocities: 3
 - velocities: (0: 0), (1: 1), (2: -1),



The domain

In order to build the domain of the simulation, the dictionary should contain the space step Δx and the stencils of the velocities (one for each scheme).

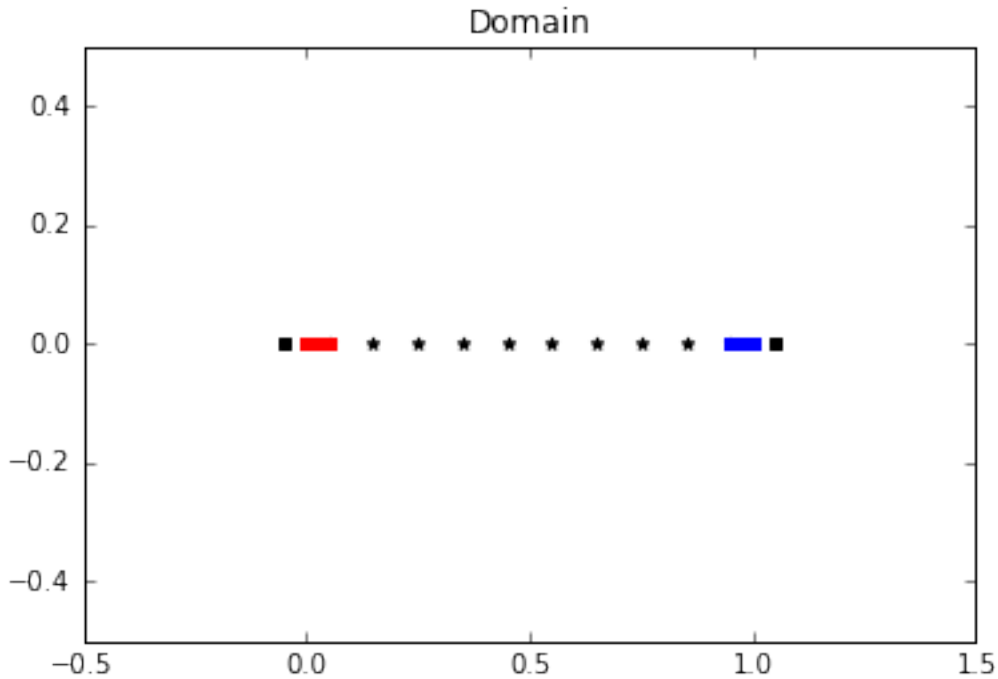
We construct a domain with $N = 10$ points in space.

```
In [4]: N = 10
        dx = (xmax-xmin)/N
        dico_dom = {
            'box': {'x': [xmin, xmax], 'label': 0},
            'space_step': dx,
            'schemes': [
                {
                    'velocities': list(range(3)),
                },
            ],
        }
        dom = pyLBM.Domain(dico_dom)
        print(dom)
        dom.visualize()
```

Domain informations

- spatial dimension: 1

space step: dx= 1.000e-01



The scheme

In pyLBM, a simulation can be performed by using several coupled schemes. In this example, a single scheme is used and defined through a list of one single dictionary. This dictionary should contain:

- ‘velocities’: a list of the velocities
- ‘conserved_moments’: a list of the conserved moments as sympy variables
- ‘polynomials’: a list of the polynomials that define the moments
- ‘equilibrium’: a list of the equilibrium value of all the moments
- ‘relaxation_parameters’: a list of the relaxation parameters (0 for the conserved moments)
- ‘init’: a dictionary to initialize the conserved moments

(see the documentation for more details)

The scheme velocity could be taken to $1/\Delta x$ and the initial value of u to

$$u(t=0, x) = \sin(\pi x).$$

```
In [5]: import sympy as sp
```

```
def solution(x, t):
    return np.sin(np.pi*x)*np.exp(-np.pi**2*mu*t)

# parameters
mu = 1.
la = 1./dx
s1 = 2./(1+2*mu)
```

```
s2 = 1.
u, X = sp.symbols('u, X')

dico_sch = {
    'dim':1,
    'scheme_velocity':1a,
    'schemes':[
        {
            'velocities':list(range(3)),
            'conserved_moments':u,
            'polynomials':[1, X, X**2/2],
            'equilibrium':[u, 0., .5*u],
            'relaxation_parameters':[0., s1, s2],
            'init':{u:(solution, (0.,))},
        }
    ],
}

sch = pyLBM.Scheme(dico_sch)
print(sch)
```

[0] WARNING pyLBM.scheme in function __init__ line 229
The value 'space_step' is not given or wrong.
The scheme takes default value: dx = 1.
WARNING:pyLBM.scheme:The value 'space_step' is not given or wrong.
The scheme takes default value: dx = 1.

Scheme informations

- spatial dimension: dim=1
- number of schemes: nscheme=1
- number of velocities:

Stencil.nv[0]=3

- velocities value:

v[0]=(0: 0), (1: 1), (2: -1),

- polynomials:

P[0]=Matrix([[1], [X], [X**2/2]])

- equilibria:

EQ[0]=Matrix([[u], [0.0], [0.5*u]])

- relaxation parameters:

s[0]=[0.0, 0.6666666666666666, 1.0]

- moments matrices

M = [Matrix([
[1, 1, 1],
[0, 1, -1],
[0, 1/2, 1/2]])]
invM = [Matrix([
[1, 0, -2],
[0, 1/2, 1],
[0, -1/2, 1]])]

The simulation

A simulation is built by defining a correct dictionary.

We combine the previous dictionaries to build a simulation. In order to impose the homogeneous Dirichlet conditions in $x = 0$ and $x = 1$, the dictionary should contain the key 'boundary_conditions' (we use pyLBM.bc.Anti_bounce_back function).

```
In [6]: dico = {
    'box':{'x':[xmin, xmax], 'label':0},
    'space_step':dx,
    'scheme_velocity':la,
    'schemes':[
        {
            'velocities':list(range(3)),
            'conserved_moments':u,
            'polynomials':[1, X, X**2/2],
            'equilibrium':[u, 0., .5*u],
            'relaxation_parameters':[0., s1, s2],
            'init':{u:(solution, (0.,))},
        }
    ],
    'boundary_conditions':{
        0:{'method':{0:pyLBM.bc.anti_bounce_back,}, 'value':None},
    },
}

sol = pyLBM.Simulation(dico)
print(sol)
```

Simulation informations:

Domain informations

spatial dimension: 1
space step: dx= 1.000e-01

Scheme informations

spatial dimension: dim=1
number of schemes: nscheme=1
number of velocities:

Stencil.nv[0]=3

velocities value:

v[0]=(0: 0), (1: 1), (2: -1),

polynomials:

P[0]=Matrix([[1], [X], [X**2/2]])

equilibria:

EQ[0]=Matrix([[u], [0.0], [0.5*u]])

relaxation parameters:

s[0]=[0.0, 0.6666666666666666, 1.0]

moments matrices

```
M = [Matrix([
[1, 1, 1],
[0, 1, -1],
[0, 1/2, 1/2]])]
invM = [Matrix([
[1, 0, -2],
[0, 1/2, 1],
[0, -1/2, 1]])]
```

Run a simulation

Once the simulation is initialized, one time step can be performed by using the function `one_time_step`.

We compute the solution of the heat equation at $t = 0.1$. And, on the same graphic, we plot the initial condition, the exact solution and the numerical solution.

```
In [7]: import numpy as np
import sympy as sp
```

```
import pylab as plt
import pyLBM

u, X = sp.symbols('u, X')

def solution(x, t):
    return np.sin(np.pi*x)*np.exp(-np.pi**2*mu*t)

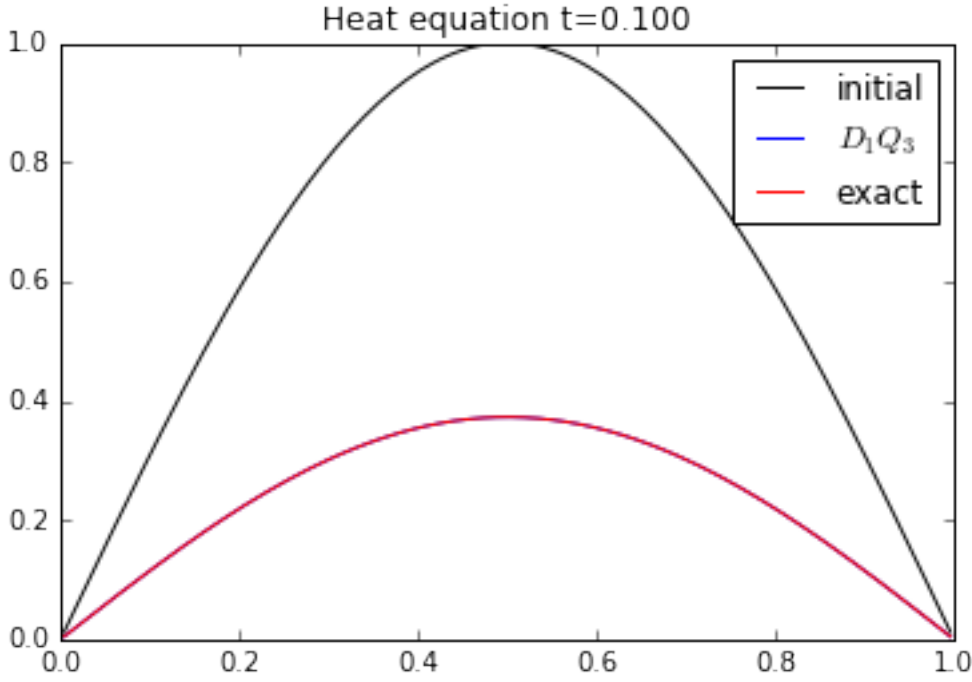
xmin, xmax = 0., 1.
N = 128
mu = 1.
Tf = .1
dx = (xmax-xmin)/N # spatial step
la = 1./dx
s1 = 2./(1+2*mu)
s2 = 1.
dico = {
    'box':{'x':[xmin,xmax], 'label':0},
    'space_step':dx,
    'scheme_velocity':la,
    'schemes':[
        {
            'velocities':list(range(3)),
            'conserved_moments':u,
            'polynomials':[1, X, X**2/2],
            'equilibrium':[u, 0., .5*u],
            'relaxation_parameters':[0., s1, s2],
            'init':{'u':(solution, (0.,))},
        }
    ],
    'boundary_conditions':{
        0:{'method':{'0':pyLBM.bc.anti_bounce_back}, 'value':None},
    },
}

sol = pyLBM.Simulation(dico)
x = sol.domain.x
y = sol.m[u]

plt.figure(1)
plt.plot(x, y, 'k', label='initial')

while sol.t < 0.1:
    sol.one_time_step()

plt.plot(x, y, 'b', label=r'$D_{1Q_3}$')
plt.plot(x, solution(x, sol.t), 'r', label='exact')
plt.title('Heat equation t={0:5.3f}'.format(sol.t))
plt.legend()
```



```
In [ ]:
```

```
In [ ]: from __future__ import print_function, division
        from six.moves import range
```

```
In [1]: %matplotlib inline
```

The heat equation in 2D

In this tutorial, we test a very classical lattice Boltzmann scheme D_2Q_5 on the heat equation.

The problem reads

$$\begin{aligned}\partial_t u &= \mu(\partial_{xx} + \partial_{yy})u, \quad t > 0, \quad (x, y) \in (0, 1)^2, \\ u(0) &= u(1) = 0,\end{aligned}$$

where μ is a constant scalar.

The scheme D_2Q_5

The numerical simulation of this equation by a lattice Boltzmann scheme consists in the approximation of the solution on discrete points of $(0, 1)^2$ at discrete instants.

To simulate this system of equations, we use the D_2Q_5 scheme given by

- five velocities $v_0 = (0, 0)$, $v_1 = (1, 0)$, $v_2 = (0, 1)$, $v_3 = (-1, 0)$, and $v_4 = (0, -1)$ with associated distribution functions f_i , $0 \leq i \leq 4$,
- a space step Δx and a time step Δt , the ratio $\lambda = \Delta x / \Delta t$ is called the scheme velocity,

- five moments

$$m_0 = \sum_{i=0}^4 f_i, \quad m_1 = \sum_{i=0}^4 v_{ix} f_i, \quad m_2 = \sum_{i=0}^4 v_{iy} f_i, \quad m_3 = \frac{1}{2} \sum_{i=0}^5 (v_{ix}^2 + v_{iy}^2) f_i, \quad m_4 = \frac{1}{2} \sum_{i=0}^5 (v_{ix}^2 - v_{iy}^2) f_i,$$

and their equilibrium values m_k^e , $0 \leq k \leq 4$. * two relaxation parameters s_1 and s_2 lying in $[0, 2]$ (s_1 for the odd moments and s_2 for the even ones).

In order to use the formalism of the package pyLBM, we introduce the five polynomials that define the moments: $P_0 = 1$, $P_1 = X$, $P_2 = Y$, $P_3 = (X^2 + Y^2)/2$, and $P_4 = (X^2 - Y^2)/2$, such that

$$m_k = \sum_{i=0}^4 P_k(v_{ix}, v_{iy}) f_i.$$

The transformation $(f_0, f_1, f_2, f_3, f_4) \mapsto (m_0, m_1, m_2, m_3, m_4)$ is invertible if, and only if, the polynomials $(P_0, P_1, P_2, P_3, P_4)$ is a free set over the stencil of velocities.

The lattice Boltzmann method consists to compute the distribution functions f_i , $0 \leq i \leq 4$ in each point of the lattice x and at each time $t^n = n\Delta t$. A step of the scheme can be read as a splitting between the relaxation phase and the transport phase:

- relaxation:

$$\begin{aligned} m_1^*(t, x, y) &= (1 - s_1) m_1(t, x, y) + s_1 m_1^e(t, x, y), \\ m_2^*(t, x, y) &= (1 - s_1) m_2(t, x, y) + s_1 m_2^e(t, x, y), \\ m_3^*(t, x, y) &= (1 - s_2) m_3(t, x, y) + s_2 m_3^e(t, x, y), \\ m_4^*(t, x, y) &= (1 - s_2) m_4(t, x, y) + s_2 m_4^e(t, x, y). \end{aligned}$$

- m2f:

$$\begin{aligned} f_0^*(t, x, y) &= m_0(t, x, y) - 2 m_3^*(t, x, y), \\ f_1^*(t, x, y) &= \frac{1}{2} (m_1^*(t, x, y) + m_3^*(t, x, y) + m_4^*(t, x, y)), \\ f_2^*(t, x, y) &= \frac{1}{2} (m_2^*(t, x, y) + m_3^*(t, x, y) - m_4^*(t, x, y)), \\ f_3^*(t, x, y) &= \frac{1}{2} (-m_1^*(t, x, y) + m_3^*(t, x, y) + m_4^*(t, x, y)), \\ f_4^*(t, x, y) &= \frac{1}{2} (-m_2^*(t, x, y) + m_3^*(t, x, y) - m_4^*(t, x, y)). \end{aligned}$$

- transport:

$$\begin{aligned} f_0(t + \Delta t, x, y) &= f_0^*(t, x, y), \\ f_1(t + \Delta t, x, y) &= f_1^*(t, x - \Delta x, y), \\ f_2(t + \Delta t, x, y) &= f_2^*(t, x, y - \Delta x), \\ f_3(t + \Delta t, x, y) &= f_3^*(t, x + \Delta x, y), \\ f_4(t + \Delta t, x, y) &= f_4^*(t, x, y + \Delta x). \end{aligned}$$

- f2m:

$$\begin{aligned} m_0(t + \Delta t, x, y) &= f_0(t + \Delta t, x, y) + f_1(t + \Delta t, x, y) + f_2(t + \Delta t, x, y) \\ &\quad + f_3(t + \Delta t, x, y) + f_4(t + \Delta t, x, y), \\ m_1(t + \Delta t, x, y) &= f_1(t + \Delta t, x, y) - f_3(t + \Delta t, x, y), \\ m_2(t + \Delta t, x, y) &= f_2(t + \Delta t, x, y) - f_4(t + \Delta t, x, y), \\ m_3(t + \Delta t, x, y) &= \frac{1}{2} (f_1(t + \Delta t, x, y) + f_2(t + \Delta t, x, y) + f_3(t + \Delta t, x, y) + f_4(t + \Delta t, x, y)), \\ m_4(t + \Delta t, x, y) &= \frac{1}{2} (f_1(t + \Delta t, x, y) - f_2(t + \Delta t, x, y) + f_3(t + \Delta t, x, y) - f_4(t + \Delta t, x, y)). \end{aligned}$$

The moment of order 0, m_0 , being conserved during the relaxation phase, a diffusive scaling $\Delta t = \Delta x^2$, yields to the following equivalent equation

$$\partial_t m_0 = \left(\frac{1}{s_1} - \frac{1}{2}\right) (\partial_{xx} (m_3^e + m_4^e) + \partial_{yy} (m_3^e - m_4^e)) + \mathcal{O}(\Delta x^2),$$

if $m_1^e = 0$. In order to be consistent with the heat equation, the following choice is done:

$$m_3^e = \frac{1}{2}u, \quad m_4^e = 0, \quad s_1 = \frac{2}{1 + 4\mu}, \quad s_2 = 1.$$

Using pyLBM

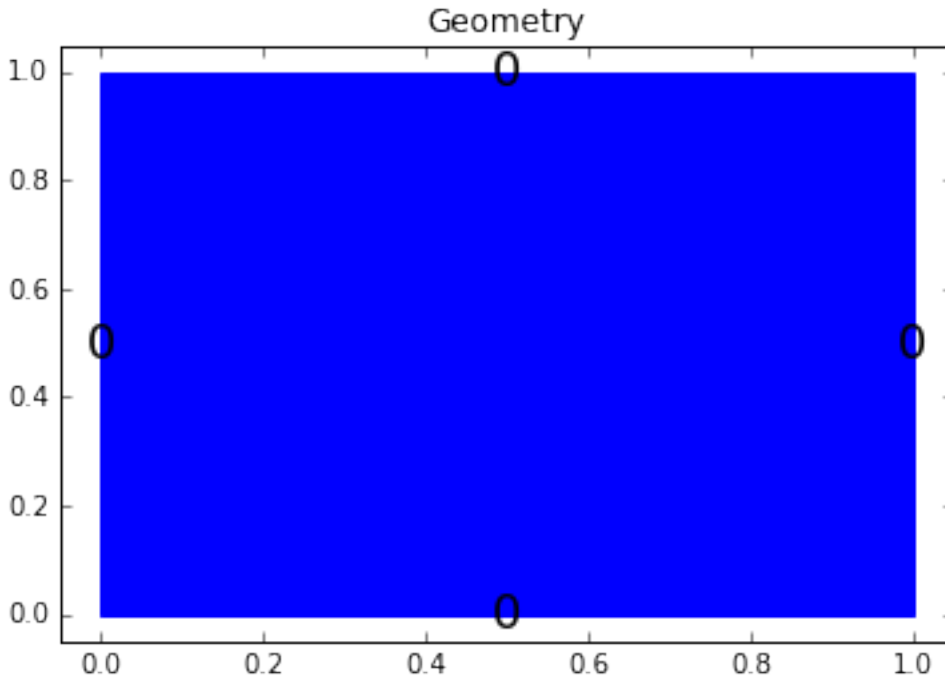
pyLBM uses Python dictionary to describe the simulation. In the following, we will build this dictionary step by step.

The geometry

In pyLBM, the geometry is defined by a box and a label for the boundaries. We define here a square $(0, 1)^2$.

```
In [2]: import pyLBM
import numpy as np
import pylab as plt
xmin, xmax, ymin, ymax = 0., 1., 0., 1.
dico_geom = {
    'box': {'x': [xmin, xmax], 'y': [ymin, ymax], 'label': 0},
}
geom = pyLBM.Geometry(dico_geom)
print(geom)
geom.visualize(viewlabel=True)

Geometry informations
  spatial dimension: 2
  bounds of the box:
[[ 0.  1.]
 [ 0.  1.]]
```



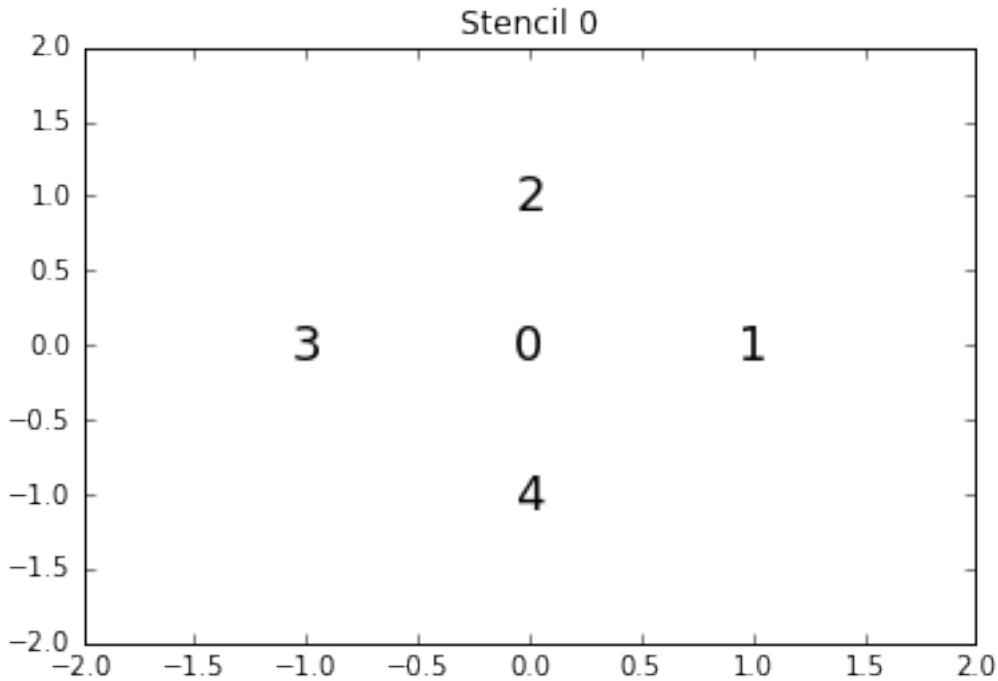
The stencil

pyLBM provides a class `stencil` that is used to define the discret velocities of the scheme. In this example, the stencil is composed by the velocities $v_0 = (0, 0)$, $v_1 = (1, 0)$, $v_2 = (-1, 0)$, $v_3 = (0, 1)$, and $v_4 = (0, -1)$ numbered by `[0, 1, 2, 3, 4]`.

```
In [3]: dico_sten = {
        'dim': 2,
        'schemes': [{'velocities': list(range(5))}],
      }
      sten = pyLBM.Stencil(dico_sten)
      print(sten)
      sten.visualize()
```

Stencil informations

```
* spatial dimension: 2
* maximal velocity in each direction: [1 1]
* minimal velocity in each direction: [-1 -1]
* Informations for each elementary stencil:
  stencil 0
    - number of velocities: 5
    - velocities: (0: 0, 0), (1: 1, 0), (2: 0, 1), (3: -1, 0), (4: 0, -1),
```

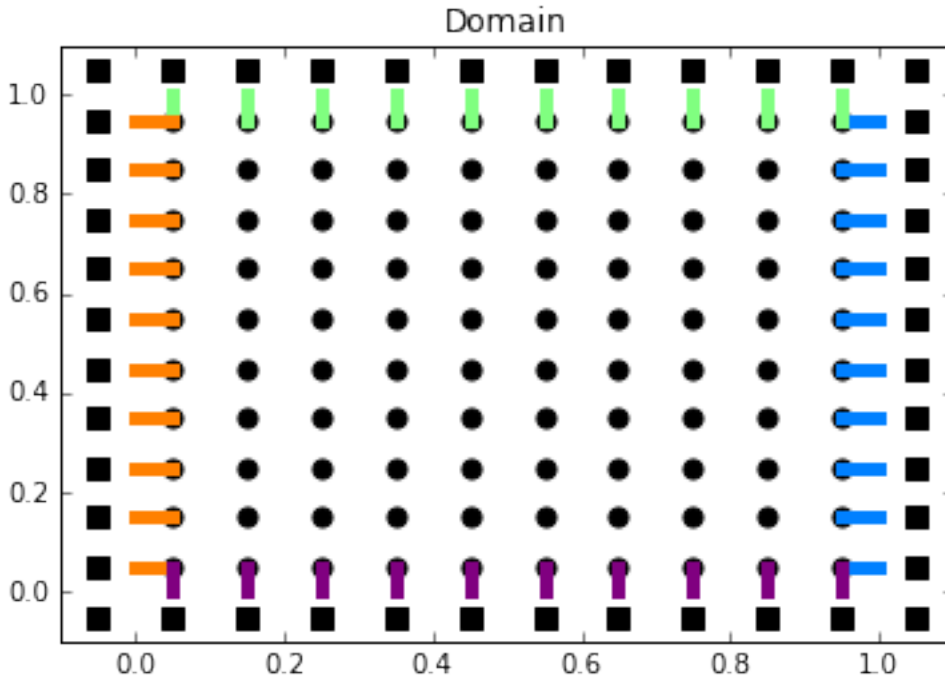
The domain

In order to build the domain of the simulation, the dictionary should contain the space step Δx and the stencils of the velocities (one for each scheme).

We construct a domain with $N = 10$ points in space.

```
In [4]: N = 10
        dx = (xmax-xmin)/N
        dico_dom = {
            'box': {'x': [xmin, xmax], 'y': [ymin, ymax], 'label': 0},
            'space_step': dx,
            'schemes': [
                {
                    'velocities': list(range(5)),
                }
            ],
        }
        dom = pyLBM.Domain(dico_dom)
        print(dom)
        dom.visualize(view_distance=True)
```

```
Domain informations
  spatial dimension: 2
  space step: dx= 1.000e-01
```



The scheme

In pyLBM, a simulation can be performed by using several coupled schemes. In this example, a single scheme is used and defined through a list of one single dictionary. This dictionary should contain:

- ‘velocities’: a list of the velocities
- ‘conserved_moments’: a list of the conserved moments as sympy variables
- ‘polynomials’: a list of the polynomials that define the moments
- ‘equilibrium’: a list of the equilibrium value of all the moments
- ‘relaxation_parameters’: a list of the relaxation parameters (0 for the conserved moments)
- ‘init’: a dictionary to initialize the conserved moments

(see the documentation for more details)

The scheme velocity could be taken to $1/\Delta x$ and the initial value of u to

$$u(t=0, x) = \sin(\pi x) \sin(\pi y).$$

In [5]: `import sympy as sp`

```
def solution(x, y, t):
    return np.sin(np.pi*x)*np.sin(np.pi*y)*np.exp(-2*np.pi**2*mu*t)

# parameters
mu = 1.
la = 1./dx
s1 = 2./(1+4*mu)
s2 = 1.
u, X, Y = sp.symbols('u, X, Y')
```

```

dico_sch = {
    'dim':2,
    'scheme_velocity':1a,
    'schemes':[
        {
            'velocities':list(range(5)),
            'conserved_moments':u,
            'polynomials':[1, X, Y, (X**2+Y**2)/2, (X**2-Y**2)/2],
            'equilibrium':[u, 0., 0., .5*u, 0.],
            'relaxation_parameters':[0., s1, s1, s2, s2],
            'init':{u:(solution, (0.,))},
        }
    ],
}

sch = pyLBM.Scheme(dico_sch)
print(sch)

```

[0] WARNING pyLBM.scheme in function __init__ line 229
The value 'space_step' is not given or wrong.
The scheme takes default value: dx = 1.
WARNING:pyLBM.scheme:The value 'space_step' is not given or wrong.
The scheme takes default value: dx = 1.

Scheme informations

```

    spatial dimension: dim=2
    number of schemes: nscheme=1
    number of velocities:
Stencil.nv[0]=5
    velocities value:
v[0]=(0: 0, 0), (1: 1, 0), (2: 0, 1), (3: -1, 0), (4: 0, -1),
    polynomials:
P[0]=Matrix([[1], [X], [Y], [X**2/2 + Y**2/2], [X**2/2 - Y**2/2]])
    equilibria:
EQ[0]=Matrix([[u], [0.0], [0.0], [0.5*u], [0.0]])
    relaxation parameters:
s[0]=[0.0, 0.4, 0.4, 1.0, 1.0]
    moments matrices
M = [Matrix([
[1, 1, 1, 1, 1],
[0, 1, 0, -1, 0],
[0, 0, 1, 0, -1],
[0, 1/2, 1/2, 1/2, 1/2],
[0, 1/2, -1/2, 1/2, -1/2]])]
invM = [Matrix([
[1, 0, 0, -2, 0],
[0, 1/2, 0, 1/2, 1/2],
[0, 0, 1/2, 1/2, -1/2],
[0, -1/2, 0, 1/2, 1/2],
[0, 0, -1/2, 1/2, -1/2]])]

```

The simulation

A simulation is built by defining a correct dictionary.

We combine the previous dictionaries to build a simulation. In order to impose the homogeneous Dirichlet conditions in $x = 0$, $x = 1$, $y = 0$, and $y = 1$, the dictionary should contain the key 'boundary_conditions' (we use

pyLBM.bc.Anti_bounce_back function).

```
In [6]: dico = {
    'box':{'x':[xmin, xmax], 'y':[ymin, ymax], 'label':0},
    'space_step':dx,
    'scheme_velocity':la,
    'schemes':[
        {
            'velocities':list(range(5)),
            'conserved_moments':u,
            'polynomials':[1, X, Y, (X**2+Y**2)/2, (X**2-Y**2)/2],
            'equilibrium':[u, 0., 0., .5*u, 0.],
            'relaxation_parameters':[0., s1, s1, s2, s2],
            'init':{u:(solution,(0.,))},
        }
    ],
    'boundary_conditions':{
        0:{'method':{0:pyLBM.bc.anti_bounce_back}, 'value':None},
    },
}

sol = pyLBM.Simulation(dico)
print(sol)
```

Simulation informations:

Domain informations

spatial dimension: 2
space step: dx= 1.000e-01

Scheme informations

spatial dimension: dim=2
number of schemes: nscheme=1
number of velocities:

Stencil.nv[0]=5

velocities value:

v[0]=(0: 0, 0), (1: 1, 0), (2: 0, 1), (3: -1, 0), (4: 0, -1),

polynomials:

P[0]=Matrix([[1], [X], [Y], [X**2/2 + Y**2/2], [X**2/2 - Y**2/2]])

equilibria:

EQ[0]=Matrix([[u], [0.0], [0.0], [0.5*u], [0.0]])

relaxation parameters:

s[0]=[0.0, 0.4, 0.4, 1.0, 1.0]

moments matrices

```
M = [Matrix([
[1, 1, 1, 1, 1],
[0, 1, 0, -1, 0],
[0, 0, 1, 0, -1],
[0, 1/2, 1/2, 1/2, 1/2],
[0, 1/2, -1/2, 1/2, -1/2]])]
invM = [Matrix([
[1, 0, 0, -2, 0],
[0, 1/2, 0, 1/2, 1/2],
[0, 0, 1/2, 1/2, -1/2],
[0, -1/2, 0, 1/2, 1/2],
[0, 0, -1/2, 1/2, -1/2]])]
```

Run a simulation

Once the simulation is initialized, one time step can be performed by using the function `one_time_step`.

We compute the solution of the heat equation at $t = 0.1$. On the same graphic, we plot the initial condition, the exact solution and the numerical solution.

```
In [9]: import numpy as np
import sympy as sp
import pylab as plt
%matplotlib inline
from mpl_toolkits.axes_grid1 import make_axes_locatable
import pyLBM

u, X, Y = sp.symbols('u, X, Y')

def solution(x, y, t, k, l):
    return np.sin(k*np.pi*x)*np.sin(l*np.pi*y)*np.exp(-(k**2+l**2)*np.pi**2*mu*t)

def plot(i, j, z, title):
    im = axarr[i,j].imshow(z)
    divider = make_axes_locatable(axarr[i, j])
    cax = divider.append_axes("right", size="20%", pad=0.05)
    cbar = plt.colorbar(im, cax=cax, format='%6.0e')
    axarr[i, j].xaxis.set_visible(False)
    axarr[i, j].yaxis.set_visible(False)
    axarr[i, j].set_title(title)

# parameters
xmin, xmax, ymin, ymax = 0., 1., 0., 1.
N = 128
mu = 1.
Tf = .1
dx = (xmax-xmin)/N # spatial step
la = 1./dx
s1 = 2./(1+4*mu)
s2 = 1.
k, l = 1, 1 # number of the wave

dico = {
    'box':{'x':[xmin, xmax], 'y':[ymin, ymax], 'label':0},
    'space_step':dx,
    'scheme_velocity':la,
    'schemes':[
        {
            'velocities':list(range(5)),
            'conserved_moments':u,
            'polynomials':[1, X, Y, (X**2+Y**2)/2, (X**2-Y**2)/2],
            'equilibrium':[u, 0., 0., .5*u, 0.],
            'relaxation_parameters':[0., s1, s1, s2, s2],
            'init':{u:(solution,(0.,k,l))},
        }
    ],
    'boundary_conditions':{
        0:{'method':{0:pyLBM.bc.anti_bounce_back}, 'value':None},
    },
    'generator':pyLBM.generator.CythonGenerator,
}
```

```
sol = pyLBM.Simulation(dico)
x = sol.domain.x
y = sol.domain.y

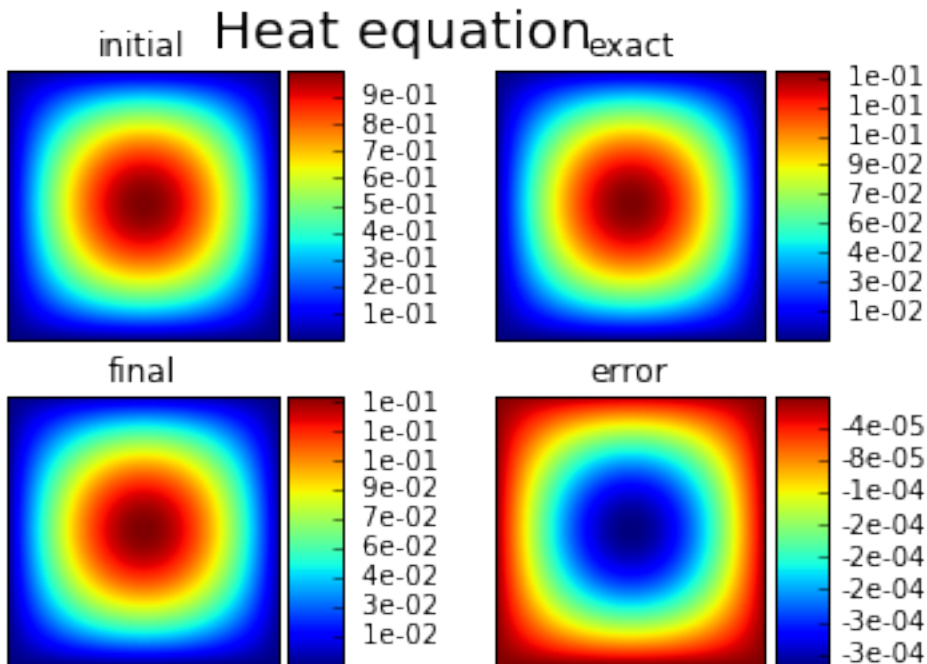
f, axarr = plt.subplots(2, 2)
f.suptitle('Heat equation', fontsize=20)

plot(0, 0, sol.m[u].copy(), 'initial')

while sol.t < Tf:
    sol.one_time_step()

sol.f2m()
z = sol.m[u]
ze = solution(x[:,np.newaxis], y[np.newaxis,:], sol.t, k, 1)
plot(1, 0, z, 'final')
plot(0, 1, ze, 'exact')
plot(1, 1, z-ze, 'error')

plt.show()
```



```
In [ ]:
```

Poiseuille flow

In this tutorial, we consider the classical D_2Q_9 to simulate a Poiseuille flow modeling by the Navier-Stokes equations.

```
In [1]: from __future__ import print_function, division
        from six.moves import range
        %matplotlib inline
```

The D_2Q_9 for Navier-Stokes

The D_2Q_9 is defined by:

- a space step Δx and a time step Δt related to the scheme velocity λ by the relation $\lambda = \Delta x / \Delta t$,
- nine velocities $\{(0, 0), (\pm 1, 0), (0, \pm 1), (\pm 1, \pm 1)\}$, identified in pyLBM by the numbers 0 to 8,
- nine polynomials used to build the moments

$$\{1, \lambda X, \lambda Y, 3E - 4, (9E^2 - 21E + 8)/2, 3XE - 5X, 3YE - 5Y, X^2 - Y^2, XY\},$$

where $E = X^2 + Y^2$.

- three conserved moments ρ, q_x , and q_y ,
- nine relaxation parameters (three are 0 corresponding to conserved moments): $\{0, 0, 0, s_\mu, s_\mu, s_\eta, s_\eta, s_\eta, s_\eta\}$, where s_μ and s_η are in $(0, 2)$,
- equilibrium value of the non conserved moments

$$\begin{aligned} m_3^e &= -2\rho + 3(q_x^2 + q_y^2)/(\rho_0\lambda^2), \\ m_4^e &= \rho - 3(q_x^2 + q_y^2)/(\rho_0\lambda^2), \\ m_5^e &= -q_x/\lambda, \\ m_6^e &= -q_y/\lambda, \\ m_7^e &= (q_x^2 - q_y^2)/(\rho_0\lambda^2), \\ m_8^e &= q_xq_y/(\rho_0\lambda^2), \end{aligned}$$

where ρ_0 is a given scalar.

This scheme is consistant at second order with the following equations (taken $\rho_0 = 1$)

$$\begin{aligned} \partial_t \rho + \partial_x q_x + \partial_y q_y &= 0, \\ \partial_t q_x + \partial_x (q_x^2 + p) + \partial_y (q_x q_y) &= \mu \partial_x (\partial_x q_x + \partial_y q_y) + \eta (\partial_{xx} + \partial_{yy}) q_x, \\ \partial_t q_y + \partial_x (q_x q_y) + \partial_y (q_y^2 + p) &= \mu \partial_y (\partial_x q_x + \partial_y q_y) + \eta (\partial_{xx} + \partial_{yy}) q_y, \\ &\text{with : } p = \rho \lambda^2 / 3. \end{aligned}$$

Build the simulation with pyLBM

In the following, we build the dictionary of the simulation step by step.

The geometry

The simulation is done on a rectangle of length L and width W . We can use $L = W = 1$.

We propose a dictionary that build the geometry of the domain. The labels of the bounds can be specified to different values for the moment.

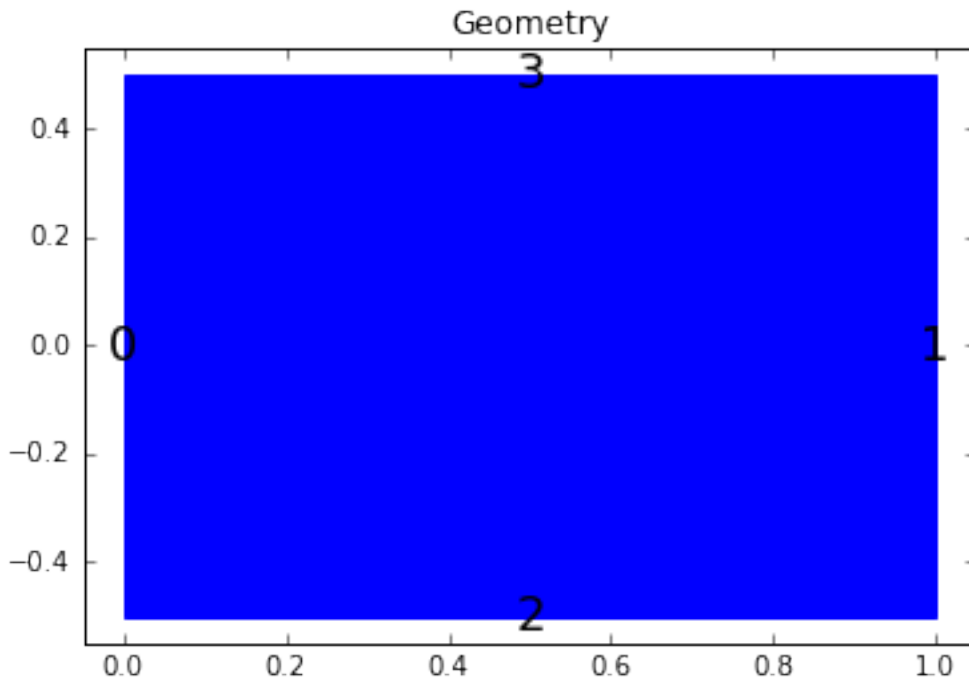
```
In [2]: import numpy as np
import matplotlib.pyplot as plt

import pyLBM

L, W = 1., 1.
dico_geom = {'box': {'x': [0, L], 'y': [-.5*W, .5*W]}, 'label': list(range(4))}
geom = pyLBM.Geometry(dico_geom)
```

```
print(geom)
geom.visualize(viewlabel=True)
```

```
Geometry informations
  spatial dimension: 2
  bounds of the box:
[[ 0.  1. ]
 [-0.5 0.5]]
```



The stencil

The stencil of the D_2Q_9 is composed by the nine following velocities in 2D:

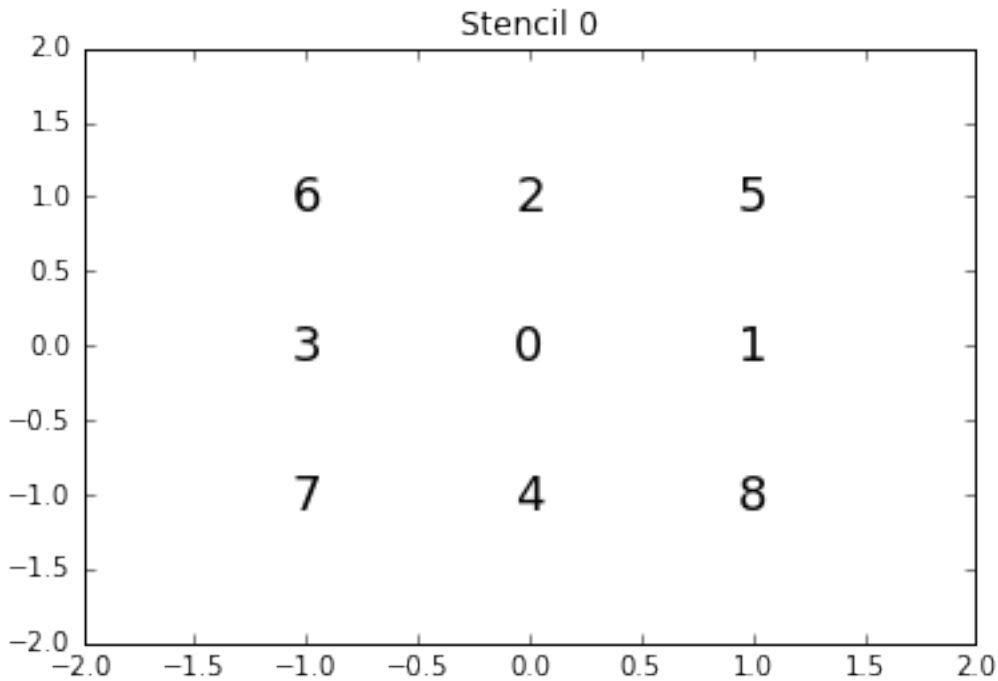
$$\begin{aligned}v_0 &= (0, 0), \\v_1 &= (1, 0), \quad v_2 = (0, 1), \quad v_3 = (-1, 0), \quad v_4 = (0, -1), \\v_5 &= (1, 1), \quad v_6 = (-1, 1), \quad v_7 = (-1, -1), \quad v_8 = (1, -1).\end{aligned}$$

```
In [3]: dico_sten = {
        'dim':2,
        'schemes': [{'velocities':list(range(9))}],
      }
sten = pyLBM.Stencil(dico_sten)
print(sten)
sten.visualize()
```

```
Stencil informations
* spatial dimension: 2
* maximal velocity in each direction: [1 1]
* minimal velocity in each direction: [-1 -1]
* Informations for each elementary stencil:
  stencil 0
    - number of velocities: 9
```



```
- velocities: (0: 0, 0), (1: 1, 0), (2: 0, 1), (3: -1, 0), (4: 0, -1), (5: 1, 1), (6: -1, 1), (7: -1, -1), (8: 1, -1)
```

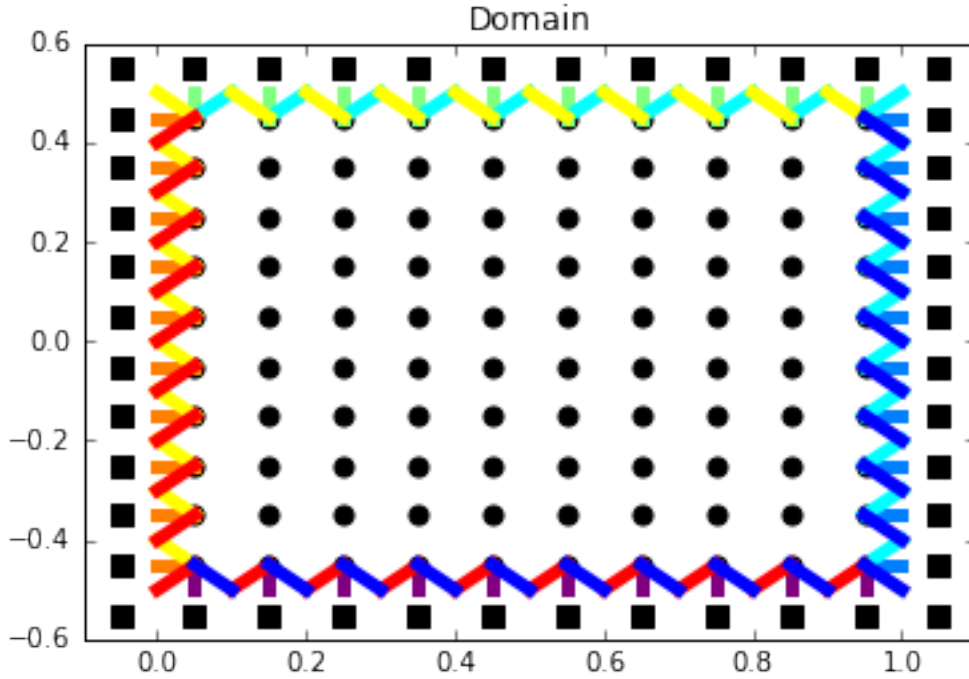


The domain

In order to build the domain of the simulation, the dictionary should contain the space step Δx and the stencils of the velocities (one for each scheme).

```
In [4]: dico_dom = {
        'space_step':.1,
        'box':{'x':[0,L], 'y':[-.5*W,.5*W], 'label':list(range(4))},
        'schemes':[{'velocities':list(range(9))}],
        }
dom = pyLBM.Domain(dico_dom)
print(dom)
dom.visualize(view_distance=True)
```

```
Domain informations
  spatial dimension: 2
  space step: dx= 1.000e-01
```



The scheme

In pyLBM, a simulation can be performed by using several coupled schemes. In this example, a single scheme is used and defined through a list of one single dictionary. This dictionary should contain:

- ‘velocities’: a list of the velocities
- ‘conserved_moments’: a list of the conserved moments as sympy variables
- ‘polynomials’: a list of the polynomials that define the moments
- ‘equilibrium’: a list of the equilibrium value of all the moments
- ‘relaxation_parameters’: a list of the relaxation parameters (0 for the conserved moments)
- ‘init’: a dictionary to initialize the conserved moments

(see the documentation for more details)

In order to fix the bulk (μ) and the shear (η) viscosities, we impose

$$s_\eta = \frac{2}{1 + \eta d}, \quad s_\mu = \frac{2}{1 + \mu d}, \quad d = \frac{6}{\lambda \rho_0 \Delta x}.$$

The scheme velocity could be taken to 1 and the initial value of ρ to $\rho_0 = 1$, q_x and q_y to 0.

In order to accelerate the simulation, we can use another generator. The default generator is Numpy (pure python). We can use for instance Cython that generates a more efficient code. This generator can be activated by using ‘generator’: pyLBM.generator.CythonGenerator in the dictionary.

```
In [5]: import sympy as sp
        X, Y, rho, qx, qy, LA = sp.symbols('X, Y, rho, qx, qy, LA')

        # parameters
        dx = 1./128 # spatial step
        la = 1.      # velocity of the scheme
```

```

L = 1          # length of the domain
W = 1          # width of the domain
rhoo = 1.      # mean value of the density
mu = 1.e-3    # shear viscosity
eta = 1.e-1    # bulk viscosity
# initialization
xmin, xmax, ymin, ymax = 0.0, L, -0.5*W, 0.5*W
dummy = 3.0/(la*rhoo*dx)
s_mu = 1.0/(0.5+mu*dummy)
s_eta = 1.0/(0.5+eta*dummy)
s_q = s_eta
s_es = s_mu
s = [0., 0., 0., s_mu, s_es, s_q, s_q, s_eta, s_eta]
dummy = 1./(LA**2*rhoo)
qx2 = dummy*qx**2
qy2 = dummy*qy**2
q2 = qx2+qy2
qxy = dummy*qx*qy

dico_sch = {
    'box':{'x':[xmin, xmax], 'y':[ymin, ymax], 'label':0},
    'space_step':dx,
    'scheme_velocity':la,
    'parameters':{'LA':la},
    'schemes':[
        {
            'velocities':list(range(9)),
            'conserved_moments':[rho, qx, qy],
            'polynomials':[
                1, LA*X, LA*Y,
                3*(X**2+Y**2)-4,
                (9*(X**2+Y**2)**2-21*(X**2+Y**2)+8)/2,
                3*X*(X**2+Y**2)-5*X, 3*Y*(X**2+Y**2)-5*Y,
                X**2-Y**2, X*Y
            ],
            'relaxation_parameters':s,
            'equilibrium':[
                rho, qx, qy,
                -2*rho + 3*q2,
                rho-3*q2,
                -qx/LA, -qy/LA,
                qx2-qy2, qxy
            ],
            'init':{'rho':rhoo, 'qx':0., 'qy':0.},
        },
    ],
    'generator': pyLBM.generator.CythonGenerator,
}
sch = pyLBM.Scheme(dico_sch)
print(sch)

```

Scheme informations

```

spatial dimension: dim=2
number of schemes: nscheme=1
number of velocities:

```

```
Stencil.nv[0]=9
```

```
velocities value:
```

```
v[0]=(0: 0, 0), (1: 1, 0), (2: 0, 1), (3: -1, 0), (4: 0, -1), (5: 1, 1), (6: -1, 1), (7: -1, -1),
```

```
polynomials:
```

```
P[0]=Matrix([[1], [LA*X], [LA*Y], [3*X**2 + 3*Y**2 - 4], [-21*X**2/2 - 21*Y**2/2 + 9*(X**2 + Y**2)],
equilibria:
EQ[0]=Matrix([[rho], [qx], [qy], [-2*rho + 3.0*qx**2/LA**2 + 3.0*qy**2/LA**2], [rho - 3.0*qx**2/LA**2 + 3.0*qy**2/LA**2]],
relaxation parameters:
s[0]=[0.0, 0.0, 0.0, 1.1312217194570136, 1.1312217194570136, 0.025706940874035987, 0.025706940874035987],
moments matrices
M = [Matrix([
[ 1, 1, 1, 1, 1, 1, 1, 1, 1],
[ 0, LA, 0, -LA, 0, LA, -LA, -LA, LA],
[ 0, 0, LA, 0, -LA, LA, LA, -LA, -LA],
[-4, -1, -1, -1, -1, 2, 2, 2, 2],
[ 4, -2, -2, -2, -2, 1, 1, 1, 1],
[ 0, -2, 0, 2, 0, 1, -1, -1, 1],
[ 0, 0, -2, 0, 2, 1, 1, -1, -1],
[ 0, 1, -1, 1, -1, 0, 0, 0, 0],
[ 0, 0, 0, 0, 0, 1, -1, 1, -1]])]
invM = [Matrix([
[1/9, 0, 0, -1/9, 1/9, 0, 0, 0, 0],
[1/9, 1/(6*LA), 0, -1/36, -1/18, -1/6, 0, 1/4, 0],
[1/9, 0, 1/(6*LA), -1/36, -1/18, 0, -1/6, -1/4, 0],
[1/9, -1/(6*LA), 0, -1/36, -1/18, 1/6, 0, 1/4, 0],
[1/9, 0, -1/(6*LA), -1/36, -1/18, 0, 1/6, -1/4, 0],
[1/9, 1/(6*LA), 1/(6*LA), 1/18, 1/36, 1/12, 1/12, 0, 1/4],
[1/9, -1/(6*LA), 1/(6*LA), 1/18, 1/36, -1/12, 1/12, 0, -1/4],
[1/9, -1/(6*LA), -1/(6*LA), 1/18, 1/36, -1/12, -1/12, 0, 1/4],
[1/9, 1/(6*LA), -1/(6*LA), 1/18, 1/36, 1/12, -1/12, 0, -1/4]])]
```

Run the simulation

For the simulation, we take

- The domain $x \in (0, L)$ and $y \in (-W/2, W/2)$, $L = 2$, $W = 1$,
- the viscosities $\mu = 10^{-2} = \eta = 10^{-2}$,
- the space step $\Delta x = 1/128$, the scheme velocity $\lambda = 1$,
- the mean density $\rho_0 = 1$.

Concerning the boundary conditions, we impose the velocity on all the edges by a bounce-back condition with a source term that reads

$$q_x(x, y) = \rho_0 v_{\max} \left(1 - \frac{4y^2}{W^2}\right), \quad q_y(x, y) = 0, \\ \text{with : } v_{\max} = 0.1.$$

We compute the solution for $t \in (0, 50)$ and we plot several slices of the solution during the simulation.

This problem has an exact solution given by

$$q_x = \rho_0 v_{\max} \left(1 - \frac{4y^2}{W^2}\right), \quad q_y = 0, \quad p = p_0 + Kx, \\ \text{where the pressure gradient : } K \text{ reads}$$

$$K = -\frac{8v_{\max}\eta}{W^2}.$$

We compute the exact and the numerical gradients of the pressure.

```

In [7]: X, Y, LA = sp.symbols('X, Y, LA')
        rho, qx, qy = sp.symbols('rho, qx, qy')

def bc(f, m, x, y):
    m[qx] = rhoo * vmax * (1.-4.*y**2/W**2)
    m[qy] = 0.

def plot_coupe(sol):
    fig, ax1 = plt.subplots()
    ax2 = ax1.twinx()
    ax1.cla()
    ax2.cla()
    mx = int(sol.domain.shape_in[0]/2)
    my = int(sol.domain.shape_in[1]/2)
    x = sol.domain.x
    y = sol.domain.y
    u = sol.m[qx] / rhoo
    for i in [0,mx,-1]:
        ax1.plot(y+x[i], u[i, :], 'b')
    for j in [0,my,-1]:
        ax1.plot(x+y[j], u[:,j], 'b')
    ax1.set_ylabel('velocity', color='b')
    for tl in ax1.get_yticklabels():
        tl.set_color('b')
    ax1.set_ylim(-.5*rhoo*vmax, 1.5*rhoo*vmax)
    p = sol.m[rho][:,my] * la**2 / 3.0
    p -= np.average(p)
    ax2.plot(x, p, 'r')
    ax2.set_ylabel('pressure', color='r')
    for tl in ax2.get_yticklabels():
        tl.set_color('r')
    ax2.set_ylim(pressure_gradient*L, -pressure_gradient*L)
    plt.title('Poiseuille flow at t = {0:f}'.format(sol.t))
    plt.draw()
    plt.pause(1.e-3)

# parameters
dx = 1./16 # spatial step
la = 1.    # velocity of the scheme
Tf = 50    # final time of the simulation
L = 2      # length of the domain
W = 1      # width of the domain
vmax = 0.1 # maximal velocity obtained in the middle of the channel
rhoo = 1.   # mean value of the density
mu = 1.e-2 # bulk viscosity
eta = 1.e-2 # shear viscosity
pressure_gradient = - vmax * 8.0 / W**2 * eta
# initialization
xmin, xmax, ymin, ymax = 0.0, L, -0.5*W, 0.5*W
dummy = 3.0/(la*rhoo*dx)
s_mu = 1.0/(0.5+mu*dummy)
s_eta = 1.0/(0.5+eta*dummy)
s_q = s_eta
s_es = s_mu
s = [0., 0., 0., s_mu, s_es, s_q, s_q, s_eta, s_eta]
dummy = 1./(LA**2*rhoo)
qx2 = dummy*qx**2
qy2 = dummy*qy**2
q2 = qx2+qy2

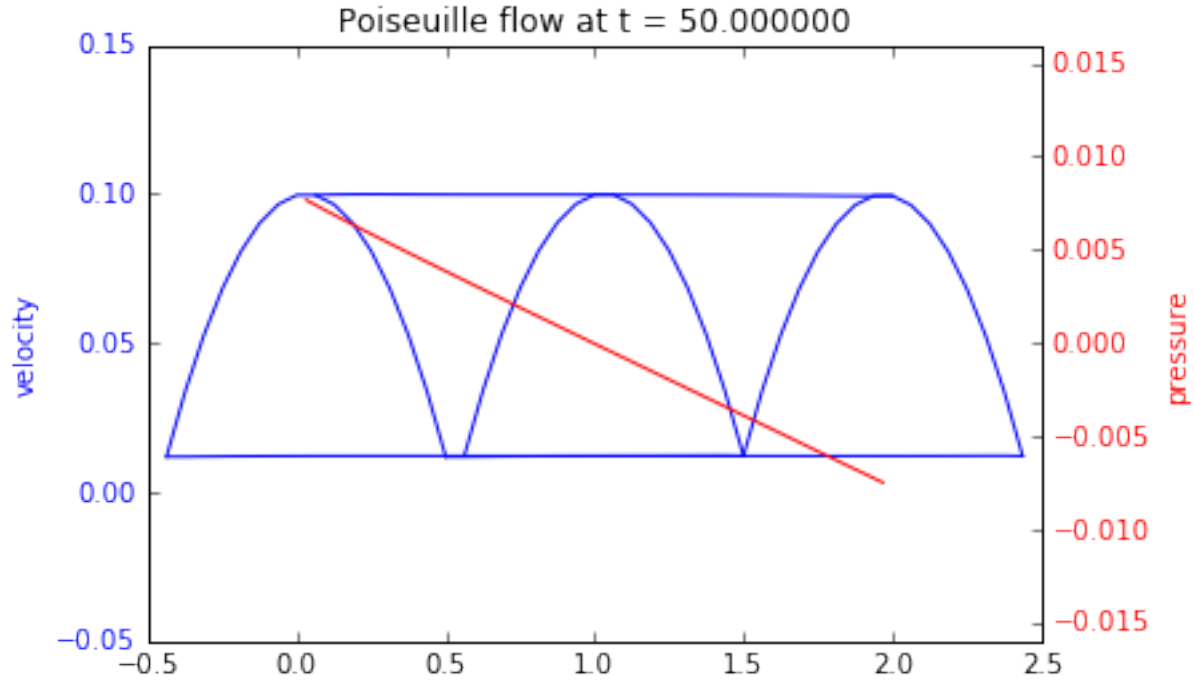
```

```
qxy = dummy*qx*qy

dico = {
    'box':{'x':[xmin, xmax], 'y':[ymin, ymax], 'label':0},
    'space_step':dx,
    'scheme_velocity':la,
    'parameters':{'LA':la},
    'schemes':[
        {
            'velocities':list(range(9)),
            'conserved_moments':[rho, qx, qy],
            'polynomials':[
                1, LA*X, LA*Y,
                3*(X**2+Y**2)-4,
                (9*(X**2+Y**2)**2-21*(X**2+Y**2)+8)/2,
                3*X*(X**2+Y**2)-5*X, 3*Y*(X**2+Y**2)-5*Y,
                X**2-Y**2, X*Y
            ],
            'relaxation_parameters':s,
            'equilibrium':[
                rho, qx, qy,
                -2*rho + 3*q2,
                rho-3*q2,
                -qx/LA, -qy/LA,
                qx2-qy2, qxy
            ],
            'init':{'rho':rho0, 'qx':0., 'qy':0.},
        },
    ],
    'boundary_conditions':{
        0:{'method':{0: pyLBM.bc.Bouzidi_bounce_back}, 'value':bc}
    },
    'generator': pyLBM.generator.CythonGenerator,
}

sol = pyLBM.Simulation(dico)
while (sol.t<Tf):
    sol.one_time_step()
plot_coupe(sol)
ny = int(sol.domain.shape_in[1]/2)
num_pressure_gradient = (sol.m[rho][-2,ny] - sol.m[rho][1,ny]) / (xmax-xmin) * la**2/ 3.0
print("Exact pressure gradient      : {0:10.3e}".format(pressure_gradient))
print("Numerical pressure gradient: {0:10.3e}".format(num_pressure_gradient))

Exact pressure gradient      : -8.000e-03
Numerical pressure gradient: -7.074e-03
```



In []:

Lid driven cavity

In this tutorial, we consider the classical D_2Q_9 and D_3Q_{15} to simulate a lid driven cavity modeling by the Navier-Stokes equations. The D_2Q_9 is used in dimension 2 and the D_3Q_{15} in dimension 3.

```
In [2]: from __future__ import print_function, division
        from six.moves import range
        %matplotlib inline
```

The D_2Q_9 for Navier-Stokes

The D_2Q_9 is defined by:

- a space step Δx and a time step Δt related to the scheme velocity λ by the relation $\lambda = \Delta x / \Delta t$,
- nine velocities $\{(0, 0), (\pm 1, 0), (0, \pm 1), (\pm 1, \pm 1)\}$, identified in pyLBM by the numbers 0 to 8,
- nine polynomials used to build the moments

$$\{1, \lambda X, \lambda Y, 3E - 4, (9E^2 - 21E + 8)/2, 3XE - 5X, 3YE - 5Y, X^2 - Y^2, XY\},$$

where $E = X^2 + Y^2$.

- three conserved moments ρ , q_x , and q_y ,
- nine relaxation parameters (three are 0 corresponding to conserved moments): $\{0, 0, 0, s_\mu, s_\mu, s_\eta, s_\eta, s_\eta, s_\eta\}$, where s_μ and s_η are in $(0, 2)$,
- equilibrium value of the non conserved moments

$$\begin{aligned}m_3^e &= -2\rho + 3(q_x^2 + q_y^2)/(\rho_0\lambda^2), \\m_4^e &= \rho - 3(q_x^2 + q_y^2)/(\rho_0\lambda^2), \\m_5^e &= -q_x/\lambda, \\m_6^e &= -q_y/\lambda, \\m_7^e &= (q_x^2 - q_y^2)/(\rho_0\lambda^2), \\m_8^e &= q_xq_y/(\rho_0\lambda^2),\end{aligned}$$

where ρ_0 is a given scalar.

This scheme is consistant at second order with the following equations (taken $\rho_0 = 1$)

$$\begin{aligned}\partial_t \rho + \partial_x q_x + \partial_y q_y &= 0, \\ \partial_t q_x + \partial_x(q_x^2 + p) + \partial_y(q_x q_y) &= \mu \partial_x(\partial_x q_x + \partial_y q_y) + \eta(\partial_{xx} + \partial_{yy})q_x, \\ \partial_t q_y + \partial_x(q_x q_y) + \partial_y(q_y^2 + p) &= \mu \partial_y(\partial_x q_x + \partial_y q_y) + \eta(\partial_{xx} + \partial_{yy})q_y,\end{aligned}$$

with $p = \rho\lambda^2/3$.

We write the dictionary for a simulation of the Navier-Stokes equations on $(0, 1)^2$.

In order to impose the boundary conditions, we use the bounce-back conditions to fix $q_x = q_y = 0$ at south, east, and west and $q_x = \rho u$, $q_y = 0$ at north. The driven velocity u could be $u = \lambda/10$.

The solution is governed by the Reynolds number $Re = \rho_0 u / \eta$. We fix the relaxation parameters to have $Re = 1000$. The relaxation parameters related to the bulk viscosity μ should be large enough to ensure the stability (for instance $\mu = 10^{-3}$).

We compute the stationary solution of the problem obtained for large enough final time. We plot the solution with the function quiver of matplotlib.

```
In [4]: import numpy as np
import sympy as sp
import matplotlib.pyplot as plt
import pyLBM

X, Y, LA = sp.symbols('X, Y, LA')
rho, qx, qy = sp.symbols('rho, qx, qy')

def bc(f, m, x, y):
    m[qx] = rhoo * vup

def plot(sol):
    pas = 2
    y, x = np.meshgrid(sol.domain.y[::pas], sol.domain.x[::pas])
    u = sol.m[qx][::pas, ::pas] / sol.m[rho][::pas, ::pas]
    v = sol.m[qy][::pas, ::pas] / sol.m[rho][::pas, ::pas]
    nv = np.sqrt(u**2 + v**2)
    normu = nv.max()
    u = u / (nv + 1e-5)
    v = v / (nv + 1e-5)
    plt.quiver(x, y, u, v, nv, pivot='mid')
    plt.title('Solution at t={0:8.2f}'.format(sol.t))
    plt.show()

# parameters
Re = 1000
dx = 1./128 # spatial step
la = 1.     # velocity of the scheme
Tf = 10     # final time of the simulation
vup = la/5  # maximal velocity obtained in the middle of the channel
```



```

rho0 = 1.      # mean value of the density
mu = 1.e-4    # bulk viscosity
eta = rho0*vup/Re # shear viscosity
# initialization
xmin, xmax, ymin, ymax = 0., 1., 0., 1.
dummy = 3.0/(1a*rho0*dx)
s_mu = 1.0/(0.5+mu*dummy)
s_eta = 1.0/(0.5+eta*dummy)
s_q = s_eta
s_es = s_mu
s = [0., 0., 0., s_mu, s_es, s_q, s_q, s_eta, s_eta]
dummy = 1./(LA**2*rho0)
qx2 = dummy*qx**2
qy2 = dummy*qy**2
q2 = qx2+qy2
qxy = dummy*qx*qy

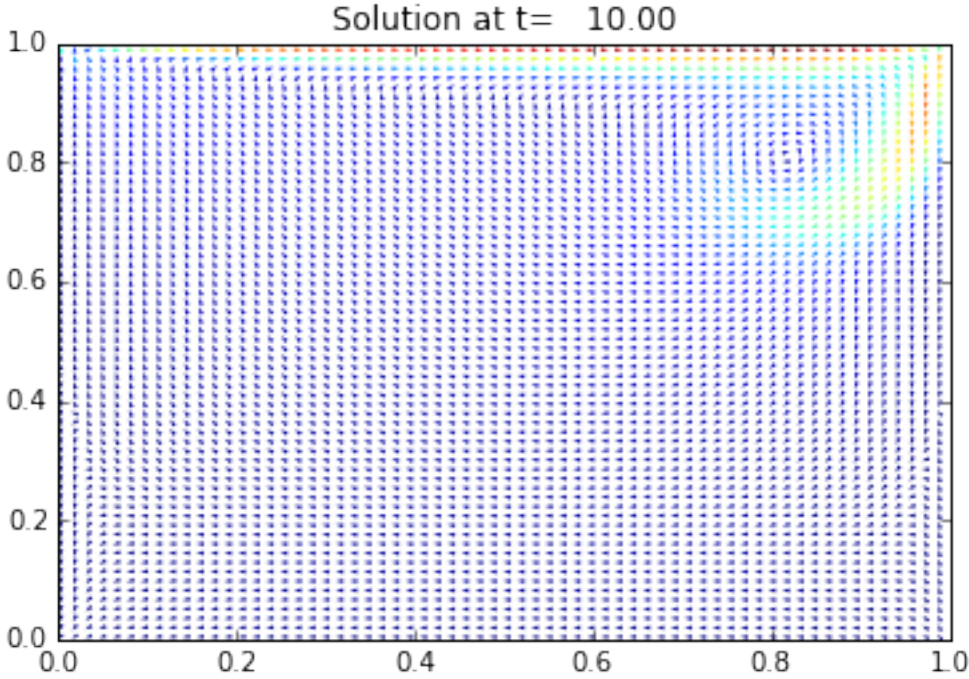
print("Reynolds number: {0:10.3e}".format(Re))
print("Bulk viscosity : {0:10.3e}".format(mu))
print("Shear viscosity: {0:10.3e}".format(eta))
print("relaxation parameters: {0}".format(s))

dico = {
    'box':{'x':[xmin, xmax], 'y':[ymin, ymax], 'label':[0,0,0,1]},
    'space_step':dx,
    'scheme_velocity':1a,
    'parameters':{'LA':1a},
    'schemes':[
        {
            'velocities':list(range(9)),
            'conserved_moments':[rho, qx, qy],
            'polynomials':[
                1, LA*X, LA*Y,
                3*(X**2+Y**2)-4,
                0.5*(9*(X**2+Y**2)**2-21*(X**2+Y**2)+8),
                3*X*(X**2+Y**2)-5*X, 3*Y*(X**2+Y**2)-5*Y,
                X**2-Y**2, X*Y
            ],
            'relaxation_parameters':s,
            'equilibrium':[
                rho, qx, qy,
                -2*rho + 3*q2,
                rho-3*q2,
                -qx/LA, -qy/LA,
                qx2-qy2, qxy
            ],
            'init':{'rho':rho0, 'qx':0., 'qy':0.},
        },
    ],
    'boundary_conditions':{
        0: {'method':{0: pyLBM.bc.Bouzidi_bounce_back}, 'value':None},
        1: {'method':{0: pyLBM.bc.Bouzidi_bounce_back}, 'value':bc}
    },
    'generator': pyLBM.generator.CythonGenerator,
}

sol = pyLBM.Simulation(dico)
while (sol.t<Tf):
    sol.one_time_step()

```

```
plot(sol)
Reynolds number: 1.000e+03
Bulk viscosity : 1.000e-04
Shear viscosity: 2.000e-04
relaxation parameters: [0.0, 0.0, 0.0, 1.8573551263001487, 1.8573551263001487, 1.7337031900138697, 1.7337031900138697, 1.7337031900138697, 1.7337031900138697, 1.7337031900138697, 1.7337031900138697, 1.7337031900138697, 1.7337031900138697, 1.7337031900138697, 1.7337031900138697]
```



The D_3Q_{15} for Navier-Stokes

The D_3Q_{15} is defined by:

- a space step Δx and a time step Δt related to the scheme velocity λ by the relation $\lambda = \Delta x / \Delta t$,
- fifteen velocities $\{(0, 0, 0), (\pm 1, 0, 0), (0, \pm 1, 0), (0, 0, \pm 1), (\pm 1, \pm 1, \pm 1)\}$, identified in pyLBM by the numbers $\{0, \dots, 6, 19, \dots, 26\}$,
- fifteen polynomials used to build the moments

$$\{1, E - 2, (15E^2 - 55E + 32)/2, X, X(5E - 13)/2, Y, Y(5E - 13)/2, Z, Z(5E - 13)/2, 3X^2 - E, Y^2 - Z^2, XY, YZ, ZX, XYZ\}$$

where $E = X^2 + Y^2 + Z^2$.

- four conserved moments ρ, q_x, q_y , and q_z ,
- fifteen relaxation parameters (four are 0 corresponding to conserved moments): $\{0, s_1, s_2, 0, s_4, 0, s_4, 0, s_4, s_9, s_9, s_{11}, s_{11}, s_{11}, s_{14}\}$,
- equilibrium value of the non conserved moments

$$\begin{aligned}
m_1^e &= -\rho + q_x^2 + q_y^2 + q_z^2, \\
m_2^e &= -\rho, \\
m_4^e &= -7q_x/3, \\
m_6^e &= -7q_y/3, \\
m_8^e &= -7q_z/3, \\
m_9^e &= (2q_x^2 - (q_y^2 + q_z^2))/3, \\
m_{10}^e &= q_y^2 - q_z^2, \\
m_{11}^e &= q_x q_y, \\
m_{12}^e &= q_y q_z, \\
m_{13}^e &= q_z q_x, \\
m_{14}^e &= 0.
\end{aligned}$$

This scheme is consistant at second order with the Navier-Stokes equations with the shear viscosity η and the relaxation parameter s_9 linked by the relation

$$s_9 = \frac{2}{1 + 6\eta/\Delta x}.$$

We write a dictionary for a simulation of the Navier-Stokes equations on $(0, 1)^3$.

In order to impose the boundary conditions, we use the bounce-back conditions to fix $q_x = q_y = q_z = 0$ at south, north, east, west, and bottom and $q_x = \rho u$, $q_y = q_z = 0$ at top. The driven velocity u could be $u = \lambda/10$.

We compute the stationary solution of the problem obtained for large enough final time. We plot the solution with the function quiver of matplotlib.

```
In [7]: X, Y, Z, LA = sp.symbols('X, Y, Z, LA')
        rho, qx, qy, qz = sp.symbols('rho, qx, qy, qz')

def bc(f, m, x, y, z):
    m[qx] = rhoo * vup

def plot(sol):
    plt.clf()
    pas = 4
    nz = int(sol.domain.shape_in[1] / 2) + 1
    y, x = np.meshgrid(sol.domain.y[:, :pas], sol.domain.x[:, :pas])
    u = sol.m[qx][:, :pas, nz, :pas] / sol.m[rho][:, :pas, nz, :pas]
    v = sol.m[qz][:, :pas, nz, :pas] / sol.m[rho][:, :pas, nz, :pas]
    nv = np.sqrt(u**2 + v**2)
    normu = nv.max()
    u = u / (nv + 1e-5)
    v = v / (nv + 1e-5)
    plt.quiver(x, y, u, v, nv, pivot='mid')
    plt.title('Solution at t={0:9.3f}'.format(sol.t))
    plt.show()

# parameters
Re = 2000
dx = 1./64 # spatial step
la = 1.    # velocity of the scheme
Tf = 3     # final time of the simulation
vup = la/10 # maximal velocity obtained in the middle of the channel
rhoo = 1.  # mean value of the density
eta = rhoo*vup/Re # shear viscosity
# initialization
xmin, xmax, ymin, ymax, zmin, zmax = 0., 1., 0., 1., 0., 1.
```

```
dummy = 3.0/(1a*rhoo*dx)

s1 = 1.6
s2 = 1.2
s4 = 1.6
s9 = 1./(.5+dummy*eta)
s11 = s9
s14 = 1.2
s = [0, s1, s2, 0, s4, 0, s4, 0, s4, s9, s9, s11, s11, s11, s14]

r = X**2+Y**2+Z**2

print("Reynolds number: {0:10.3e}".format(Re))
print("Shear viscosity: {0:10.3e}".format(eta))

dico = {
    'box':{
        'x':[xmin, xmax],
        'y':[ymin, ymax],
        'z':[zmin, zmax],
        'label':[0,0,0,0,0,1]
    },
    'space_step':dx,
    'scheme_velocity':1a,
    'parameters':{LA:1a},
    'schemes':[
        {
            'velocities':list(range(7)) + list(range(19,27)),
            'conserved_moments':[rho, qx, qy, qz],
            'polynomials':[
                1,
                r - 2, .5*(15*r**2-55*r+32),
                X, .5*(5*r-13)*X,
                Y, .5*(5*r-13)*Y,
                Z, .5*(5*r-13)*Z,
                3*X**2-r, Y**2-Z**2,
                X*Y, Y*Z, Z*X,
                X*Y*Z
            ],
            'relaxation_parameters':s,
            'equilibrium':[
                rho,
                -rho + qx**2 + qy**2 + qz**2,
                -rho,
                qx,
                -7./3*qx,
                qy,
                -7./3*qy,
                qz,
                -7./3*qz,
                1./3*(2*qx**2-(qy**2+qz**2)),
                qy**2-qz**2,
                qx*qy,
                qy*qz,
                qz*qx,
                0
            ],
            'init':{rho:rhoo, qx:0., qy:0., qz:0.},
        },
    ],
}
```

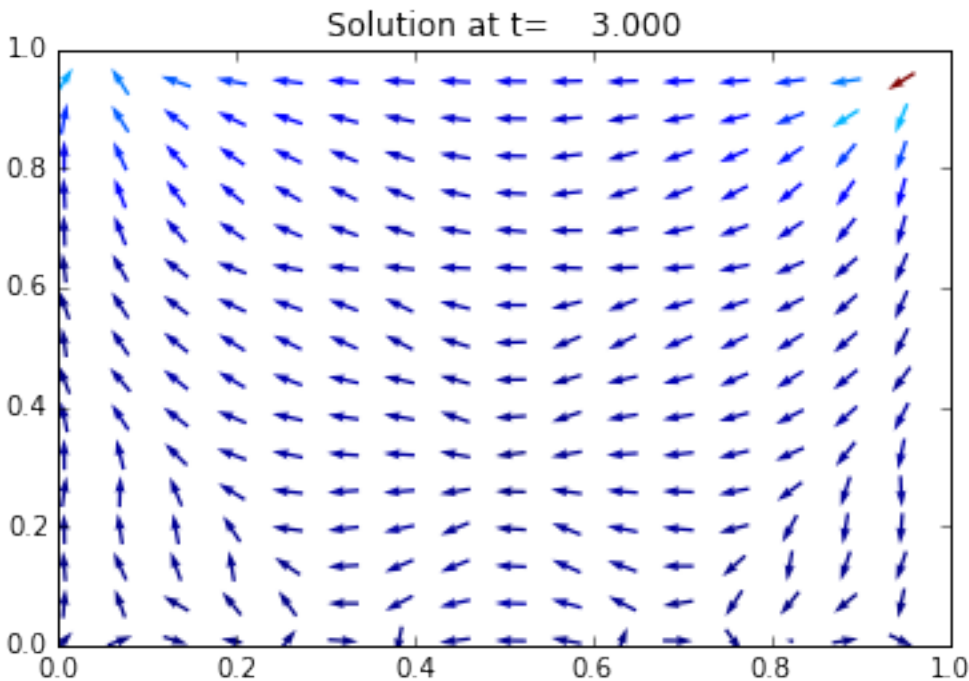
```

    ],
    'boundary_conditions':{
        0:{'method':{0: pyLBM.bc.Bouzidi_bounce_back}, 'value':None},
        1:{'method':{0: pyLBM.bc.Bouzidi_bounce_back}, 'value':bc}
    },
    'generator': pyLBM.generator.CythonGenerator,
}

sol = pyLBM.Simulation(dico)
while (sol.t<Tf):
    sol.one_time_step()
plot(sol)

```

Reynolds number: 2.000e+03
 Shear viscosity: 5.000e-05



In []:

Von Karman vortex street

In this tutorial, we consider the classical D_2Q_9 to simulate the Von Karman vortex street modeling by the Navier-Stokes equations.

In fluid dynamics, a Von Karman vortex street is a repeating pattern of swirling vortices caused by the unsteady separation of flow of a fluid around blunt bodies. It is named after the engineer and fluid dynamicist Theodore von Karman. For the simulation, we propose to simulate the Navier-Stokes equation into a rectangular domain with a circular hole of diameter d .

The D_2Q_9 is defined by:

- a space step Δx and a time step Δt related to the scheme velocity λ by the relation $\lambda = \Delta x / \Delta t$,
- nine velocities $\{(0, 0), (\pm 1, 0), (0, \pm 1), (\pm 1, \pm 1)\}$, identified in pyLBM by the numbers 0 to 8,

- nine polynomials used to build the moments

$$\{1, \lambda X, \lambda Y, 3E - 4, (9E^2 - 21E + 8)/2, 3XE - 5X, 3YE - 5Y, X^2 - Y^2, XY\},$$

where $E = X^2 + Y^2$.

- three conserved moments ρ , q_x , and q_y ,
- nine relaxation parameters (three are 0 corresponding to conserved moments): $\{0, 0, 0, s_\mu, s_\mu, s_\eta, s_\eta, s_\eta, s_\eta\}$, where s_μ and s_η are in $(0, 2)$,
- equilibrium value of the non conserved moments

$$m_3^e = -2\rho + 3(q_x^2 + q_y^2)/(\rho_0\lambda^2),$$

$$m_4^e = \rho - 3(q_x^2 + q_y^2)/(\rho_0\lambda^2),$$

$$m_5^e = -q_x/\lambda,$$

$$m_6^e = -q_y/\lambda,$$

$$m_7^e = (q_x^2 - q_y^2)/(\rho_0\lambda^2),$$

$$m_8^e = q_x q_y / (\rho_0 \lambda^2),$$

where ρ_0 is a given scalar.

This scheme is consistant at second order with the following equations (taken $\rho_0 = 1$)

$$\begin{aligned}\partial_t \rho + \partial_x q_x + \partial_y q_y &= 0, \\ \partial_t q_x + \partial_x (q_x^2 + p) + \partial_y (q_x q_y) &= \mu \partial_x (\partial_x q_x + \partial_y q_y) + \eta (\partial_{xx} + \partial_{yy}) q_x, \\ \partial_t q_y + \partial_x (q_x q_y) + \partial_y (q_y^2 + p) &= \mu \partial_y (\partial_x q_x + \partial_y q_y) + \eta (\partial_{xx} + \partial_{yy}) q_y,\end{aligned}$$

with $p = \rho\lambda^2/3$.

We write a dictionary for a simulation of the Navier-Stokes equations on $(0, 1)^2$.

In order to impose the boundary conditions, we use the bounce-back conditions to fix $q_x = q_y = \rho v_0$ at south, east, and north where the velocity v_0 could be $v_0 = \lambda/20$. At west, we impose the simple output condition of Neumann by repeating the second to last cells into the last cells.

The solution is governed by the Reynolds number $Re = \rho_0 v_0 d / \eta$, where d is the diameter of the circle. Fix the relaxation parameters to have $Re = 500$. The relaxation parameters related to the bulk viscosity μ should be large enough to ensure the stability (for instance $\mu = 10^{-3}$).

We compute the stationary solution of the problem obtained for large enough final time. We plot the vorticity of the solution with the function `imshow` of `matplotlib`.

```
In [1]: from __future__ import print_function, division
        from six.moves import range
        %matplotlib inline

In [6]: import numpy as np
        import sympy as sp
        import pyLBM

        X, Y, LA = sp.symbols('X, Y, LA')
        rho, qx, qy = sp.symbols('rho, qx, qy')

        def bc_in(f, m, x, y):
            m[qx] = rho * v0

        def vorticity(sol):
            ux = sol.m[qx] / sol.m[rho]
            uy = sol.m[qy] / sol.m[rho]
```

```

V = np.abs(uy[2:,1:-1] - uy[0:-2,1:-1] - ux[1:-1,2:] + ux[1:-1,0:-2])/(2*sol.domain.dx)
return -V

# parameters
rayon = 0.05
Re = 500
dx = 1./64 # spatial step
la = 1. # velocity of the scheme
Tf = 75 # final time of the simulation
v0 = la/20 # maximal velocity obtained in the middle of the channel
rhoo = 1. # mean value of the density
mu = 1.e-3 # bulk viscosity
eta = rhoo*v0*2*rayon/Re # shear viscosity
# initialization
xmin, xmax, ymin, ymax = 0., 3., 0., 1.
dummy = 3.0/(la*rhoo*dx)
s_mu = 1.0/(0.5+mu*dummy)
s_eta = 1.0/(0.5+eta*dummy)
s_q = s_eta
s_es = s_mu
s = [0.,0.,0.,s_mu,s_es,s_q,s_q,s_eta,s_eta]
dummy = 1./(LA**2*rhoo)
qx2 = dummy*qx**2
qy2 = dummy*qy**2
q2 = qx2+qy2
qxy = dummy*qx*qy

print("Reynolds number: {0:10.3e}".format(Re))
print("Bulk viscosity : {0:10.3e}".format(mu))
print("Shear viscosity: {0:10.3e}".format(eta))
print("relaxation parameters: {0}".format(s))

dico = {
    'box':{'x':[xmin, xmax], 'y':[ymin, ymax], 'label':[0,2,0,0]},
    'elements':[pyLBM.Circle([.3, 0.5*(ymin+ymax)+dx], rayon, label=1)],
    'space_step':dx,
    'scheme_velocity':la,
    'parameters':{'LA':la},
    'schemes':[
        {
            'velocities':list(range(9)),
            'conserved_moments':[rho, qx, qy],
            'polynomials':[
                1, LA*X, LA*Y,
                3*(X**2+Y**2)-4,
                (9*(X**2+Y**2)**2-21*(X**2+Y**2)+8)/2,
                3*X*(X**2+Y**2)-5*X, 3*Y*(X**2+Y**2)-5*Y,
                X**2-Y**2, X*Y
            ],
            'relaxation_parameters':s,
            'equilibrium':[
                rho, qx, qy,
                -2*rho + 3*q2,
                rho-3*q2,
                -qx/LA, -qy/LA,
                qx2-qy2, qxy
            ],
            'init':{'rho':rhoo, 'qx':0., 'qy':0.},
        },
    ],
}

```

```

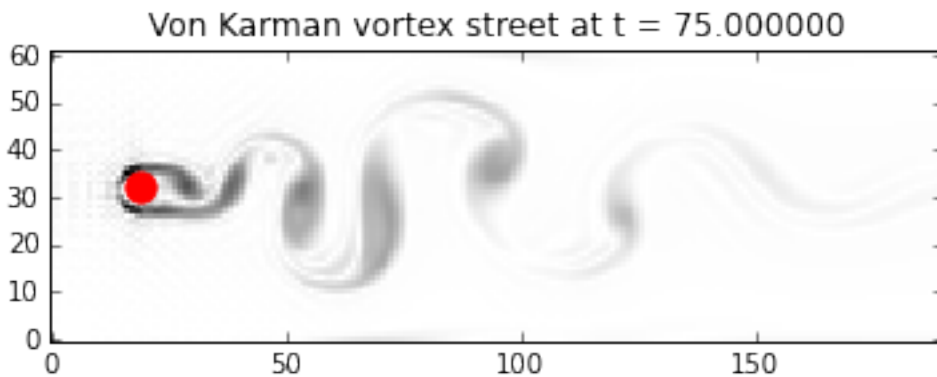
    ],
    'boundary_conditions':{
        0:{'method':{0: pyLBM.bc.Bouzidi_bounce_back}, 'value':bc_in},
        1:{'method':{0: pyLBM.bc.Bouzidi_bounce_back}, 'value':None},
        2:{'method':{0: pyLBM.bc.Neumann_x}, 'value':None},
    },
    'generator': pyLBM.generator.CythonGenerator,
}

sol = pyLBM.Simulation(dico)
while sol.t < Tf:
    sol.one_time_step()

viewer = pyLBM.viewer.matplotlibViewer
fig = viewer.Fig()
ax = fig[0]
im = ax.image(vorticity(sol).transpose(), clim = [-3., 0])
ax.ellipse([.3/dx, 0.5*(ymin+ymax)/dx], [rayon/dx, rayon/dx], 'r')
ax.title = 'Von Karman vortex street at t = {0:f}'.format(sol.t)
fig.show()

Reynolds number: 5.000e+02
Bulk viscosity : 1.000e-03
Shear viscosity: 1.000e-05
relaxation parameters: [0.0, 0.0, 0.0, 1.4450867052023122, 1.4450867052023122, 1.9923493783869939, 1

```



In []:

Transport equation with source term

In this tutorial, we propose to add a source term in the advection equation. The problem reads

$$\partial_t u + c \partial_x u = S(t, x, u), \quad t > 0, \quad x \in (0, 1),$$

where c is a constant scalar (typically $c = 1$). Additional boundary and initial conditions will be given in the following. S is the source term that can depend on the time t , the space x and the solution u .

In order to simulate this problem, we use the D_1Q_2 scheme and we add an additional `key:value` in the dictionary for the source term. We deal with two examples.

A friction term

In this example, we takes $S(t, x, u) = -\alpha u$ where α is a positive constant. The dictionary of the simulation then reads:

```
In [1]: from __future__ import print_function, division
        %matplotlib inline
        import sympy as sp
        import numpy as np
        import pyLBM

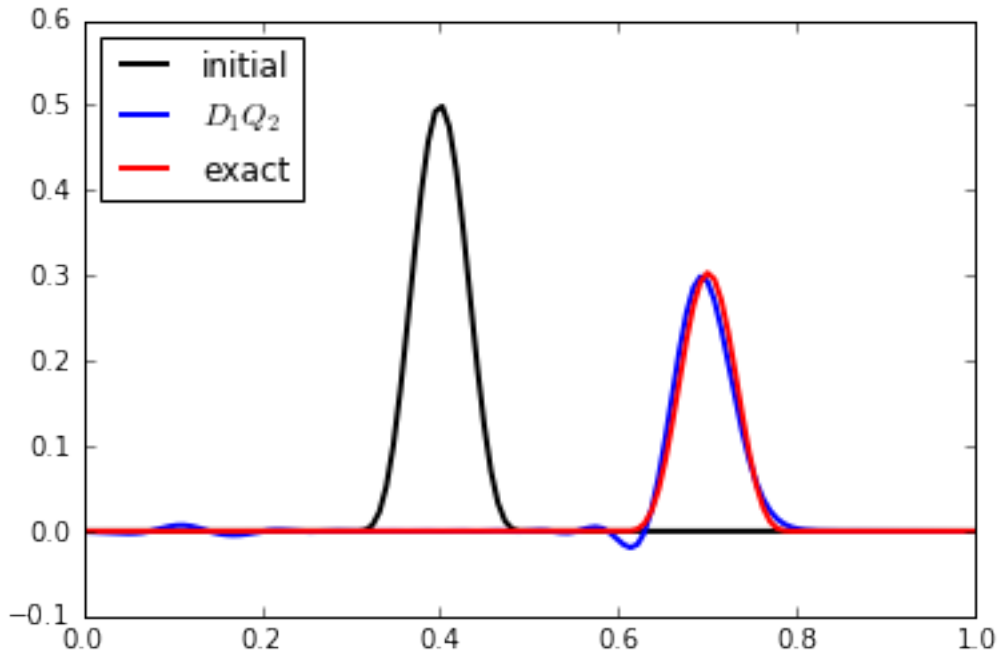
In [5]: C, ALPHA, X, u, LA = sp.symbols('C, ALPHA, X, u, LA')
        c = 0.3
        alpha = 0.5

        def init(x):
            middle, width, height = 0.4, 0.1, 0.5
            return height/width**10 * (x%1-middle-width)**5 * \
                (middle-x%1-width)**5 * (abs(x%1-middle)<=width)

        def solution(t, x):
            return init(x - c*t)*np.exp(-alpha*t)

dico = {
    'box':{'x':[0., 1.], 'label':-1},
    'space_step':1./128,
    'scheme_velocity':LA,
    'schemes':[
        {
            'velocities':[1,2],
            'conserved_moments':u,
            'polynomials':[1,LA*X],
            'relaxation_parameters':[0., 2.],
            'equilibrium':[u, C*u],
            'source_terms':{'u':-ALPHA*u},
            'init':{'u':(init,)},
        },
    ],
    'parameters': {LA: 1., C: c, ALPHA: alpha},
}

sol = pyLBM.Simulation(dico) # build the simulation
viewer = pyLBM.viewer.matplotlibViewer
fig = viewer.Fig()
ax = fig[0]
ax.axis(0., 1., -.1, .6)
x = sol.domain.x
ax.plot(x, sol.m[u], width=2, color='k', label='initial')
while sol.t < 1:
    sol.one_time_step()
    ax.plot(x, sol.m[u], width=2, color='b', label=r'$D_{1Q_2}$')
    ax.plot(x, solution(sol.t, x), width=2, color='r', label='exact')
ax.legend()
```



A source term depending on time and space

If the source term S depends explicitly on the time or on the space, we have to specify the corresponding variables in the dictionary through the key *parameters*. The time variable is prescribed by the key *'time'*. Moreover, sympy functions can be used to define the source term like in the following example. This example is just for testing the feature... no physical meaning in mind !

```
In [4]: t, C, X, u, LA = sp.symbols('t, C, X, u, LA')
        c = 0.3

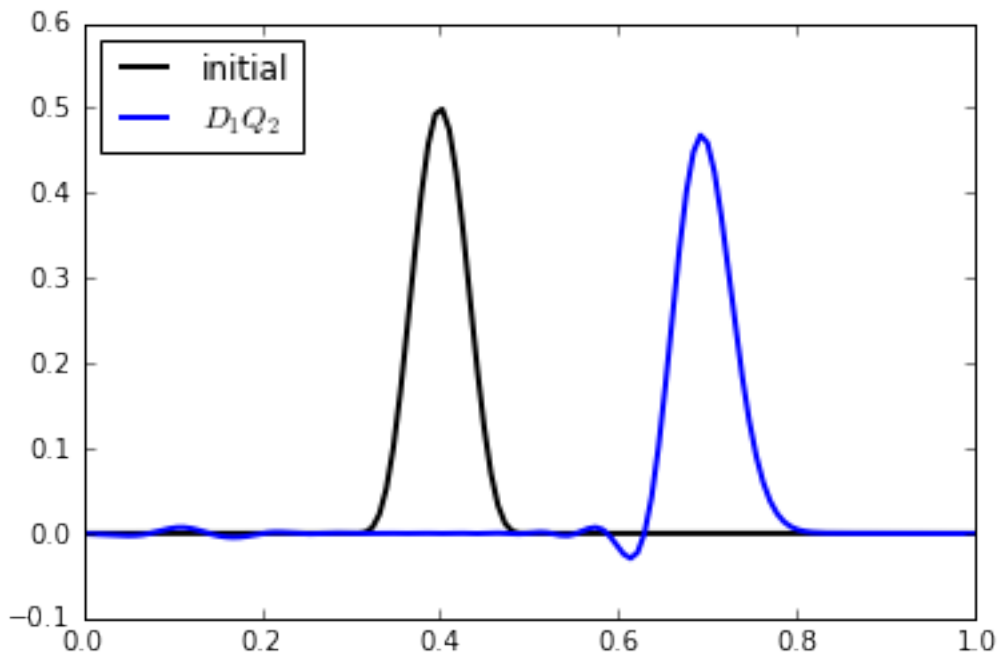
def init(x):
    middle, width, height = 0.4, 0.1, 0.5
    return height/width**10 * (x%1-middle-width)**5 * \
        (middle-x%1-width)**5 * (abs(x%1-middle)<=width)

dico = {
    'box':{'x':[0., 1.], 'label':-1},
    'space_step':1./128,
    'scheme_velocity':LA,
    'schemes':[
        {
            'velocities':[1,2],
            'conserved_moments':u,
            'polynomials':[1,LA*X],
            'relaxation_parameters':[0., 2.],
            'equilibrium':[u, C*u],
            'source_terms':{'u':-sp.Abs(X-t)**2*u},
            'init':{'u':(init,)},
        },
    ],
    'generator':pyLBM.generator.CythonGenerator,
    'parameters': {LA: 1., C: c, 'time': t},
}
```

```

sol = pyLBM.Simulation(dico) # build the simulation
viewer = pyLBM.viewer.matplotlibViewer
fig = viewer.Fig()
ax = fig[0]
ax.axis(0., 1., -.1, .6)
x = sol.domain.x
ax.plot(x, sol.m[u], width=2, color='k', label='initial')
while sol.t < 1:
    sol.one_time_step()
ax.plot(x, sol.m[u], width=2, color='b', label=r'$D_1Q_2$')
ax.legend()

```



In []:

get the notebook

Transport in 1D

In this tutorial, we will show how to implement from scratch a very basic lattice Boltzmann scheme: the D_1Q_2 for the advection equation and for Burger's equation.

get the notebook

The wave equation in 1D

In this tutorial, we will show how to implement from scratch a very basic lattice Boltzmann scheme: the D_1Q_3 for the waves equation.

get the notebook

The heat equation in 1D

In this tutorial, we present the D_1Q_3 to solve the heat equation in 1D by using pyLBM.

get the notebook

The heat equation in 2D

In this tutorial, we present the D_2Q_5 to solve the heat equation in 2D by using pyLBM.

[get the notebook](#)

Poiseuille flow

In this tutorial, we present the D_2Q_9 for Navier-Stokes equation to solve the Poiseuille flow in 2D by using pyLBM.

[get the notebook](#)

Lid driven cavity

In this tutorial, we present the D_2Q_9 for Navier-Stokes equation to solve the lid driven cavity in 2D and the $D3Q15$ in 3D by using pyLBM.

[get the notebook](#)

Von Karman vortex street

In this tutorial, we present the D_2Q_9 for Navier-Stokes equation to solve the Von Karman vortex street in 2D by using pyLBM.

[get the notebook](#)

Transport equation with source term

In this tutorial, we will show how to implement with pyLBM the D_1Q_2 for the advection equation with a source term.

Gallery

Documentation of the code

The most important classes

<i>Geometry</i> (dico)	Create a geometry that defines the fluid part and the solid part.
<i>Domain</i> ([dico, geometry, stencil, ...])	Create a domain that defines the fluid part and the solid part and computes the distances between these two states.
<i>Scheme</i> (dico[, stencil])	Create the class with all the needed informations for each elementary scheme.
<i>Simulation</i> (dico[, domain, scheme, sorder, dtype])	create a class simulation

pyLBM.Geometry

class `pyLBM.Geometry` (*dico*)

Create a geometry that defines the fluid part and the solid part.

Parameters **dico** : a dictionary that contains the following *key:value*

- **box** : a dictionary for the definition of the computed box
- **elements** : a list of elements (optional)

Notes

The dictionary that defines the box should contains the following *key:value*

- **x** : a list of the bounds in the first direction
- **y** : a list of the bounds in the second direction (optional)
- **z** : a list of the bounds in the third direction (optional)
- **label** : an integer or a list of integers (length twice the number of dimensions) used to label each edge (optional)

Examples

see `demo/examples/geometry/`

Attributes

<code>dim</code>	(int) number of spatial dimensions (1, 2, or 3)
<code>bounds</code>	(numpy array) the bounds of the box in each spatial direction
<code>box_label</code>	(list of integers) a list of the four labels for the left, right, bottom, top, front, and back edges
<code>list_elem</code>	(list of elements) a list that contains each element added or deleted in the box

Methods

<code>add_elem(elem)</code>	add a solid or a fluid part in the domain
<code>list_of_elements_labels()</code>	Get the list of all the labels used in the geometry.
<code>list_of_labels()</code>	Get the list of all the labels used in the geometry.
<code>visualize([viewer_app, viewlabel, fluid_color])</code>	plot a view of the geometry

pyLBM.Geometry.add_elem

`Geometry.add_elem(elem)`
add a solid or a fluid part in the domain

Parameters `elem` : a geometric element to add (or to del)

pyLBM.Geometry.list_of_elements_labels

`Geometry.list_of_elements_labels()`
Get the list of all the labels used in the geometry.

pyLBM.Geometry.list_of_labels

`Geometry.list_of_labels()`
Get the list of all the labels used in the geometry.

pyLBM.Geometry.visualize

`Geometry.visualize(viewer_app=<module 'pyLBM.viewer.matplotlibViewer' from
'/home/docs/checkouts/readthedocs.org/user_builds/pylbm-
loic/conda/fix-readthedoc/lib/python2.7/site-packages/pyLBM-0.3.0-
py2.7.egg/pyLBM/viewer/matplotlibViewer.pyc'>, viewlabel=False,
fluid_color='blue')`
plot a view of the geometry

Parameters `viewer_app` : a viewer (default matplotlibViewer)

viewlabel : boolean to activate the labels mark (default False)

fluid_color : color for the fluid part (default blue)

pyLBM.Domain

class `pyLBM.Domain` (*dico=None, geometry=None, stencil=None, space_step=None, verif=True*)

Create a domain that defines the fluid part and the solid part and computes the distances between these two states.

Parameters *dico* : a dictionary that contains the following *key:value*

- *box* : a dictionary that defines the computational box
- *elements* : the list of the elements (available elements are given in the module `elements`)
- *space_step* : the spatial step
- *schemes* : a list of dictionaries, each of them defining a elementary *Scheme*

Warning: the sizes of the box must be a multiple of the space step *dx*

Notes

The dictionary that defines the box should contains the following *key:value*

- *x* : a list of the bounds in the first direction
- *y* : a list of the bounds in the second direction (optional)
- *z* : a list of the bounds in the third direction (optional)
- *label* : an integer or a list of integers (length twice the number of dimensions) used to label each edge (optional)

See *Geometry* for more details.

If the geometry and/or the stencil were previously generated, it can be used directly as following

```
>>> Domain(dico, geometry = geom, stencil = sten)
```

where *geom* is an object of the class *Geometry* and *sten* an object of the class *Stencil* In that case, *dico* does not need to contain the informations for generate the geometry and/or the stencil

In 1D, `distance[q, i]` is the distance between the point `x[i]` and the border in the direction of the *q*th velocity.

In 2D, `distance[q, j, i]` is the distance between the point `(x[i], y[j])` and the border in the direction of *q*th velocity

In 3D, `distance[q, k, j, i]` is the distance between the point `(x[i], y[j], z[k])` and the border in the direction of *q*th velocity

In 1D, `flag[q, i]` is the flag of the border reached by the point `x[i]` in the direction of the *q*th velocity

In 2D, `flag[q, j, i]` is the flag of the border reached by the point `(x[i], y[j])` in the direction of *q*th velocity

In 2D, `flag[q, k, j, i]` is the flag of the border reached by the point `(x[i], y[j], z[k])` in the direction of *q*th velocity

Examples

see `demo/examples/domain/`

Attributes

<code>x</code>	x component of the coordinates in the interior domain.
<code>y</code>	y component of the coordinates in the interior domain.
<code>z</code>	z component of the coordinates in the interior domain.
<code>x_halo</code>	x component of the coordinates of the whole domain (halo points included).
<code>y_halo</code>	y component of the coordinates of the whole domain (halo points included).
<code>z_halo</code>	z component of the coordinates of the whole domain (halo points included).
<code>shape_halo</code>	shape of the whole domain with the halo points.
<code>shape_in</code>	shape of the interior domain.

pyLBM.Domain.x

`Domain.x`
x component of the coordinates in the interior domain.

pyLBM.Domain.y

`Domain.y`
y component of the coordinates in the interior domain.

pyLBM.Domain.z

`Domain.z`
z component of the coordinates in the interior domain.

pyLBM.Domain.x_halo

`Domain.x_halo`
x component of the coordinates of the whole domain (halo points included).

pyLBM.Domain.y_halo

`Domain.y_halo`
y component of the coordinates of the whole domain (halo points included).

pyLBM.Domain.z_halo

`Domain.z_halo`
z component of the coordinates of the whole domain (halo points included).

pyLBM.Domain.shape_halo

`Domain.shape_halo`

shape of the whole domain with the halo points.

pyLBM.Domain.shape_in

`Domain.shape_in`

shape of the interior domain.

<code>dim</code>	(int) number of spatial dimensions (example: 1, 2, or 3)
<code>global-bounds</code>	(numpy array) the bounds of the box in each spatial direction
<code>bounds</code>	(numpy array) the local bounds of the process in each spatial direction
<code>dx</code>	(double) space step (example: 0.1, 1.e-3)
<code>type</code>	(string) type of data (example: 'float64')
<code>stencil</code>	the stencil of the velocities (object of the class <i>Stencil</i>)
<code>:</code>	
<code>global_size</code>	(list of int) number of points in each direction
<code>extent</code>	(list of int) number of points to add on each side (max velocities)
<code>coords</code>	(numpy array) coordinates of the domain
<code>in_or_out</code>	(numpy array) defines the fluid and the solid part (fluid: value=valin, solid: value=valout)
<code>distance</code>	(numpy array) defines the distances to the borders. The distance is scaled by dx and is not equal to valin only for the points that reach the border with the specified velocity.
<code>flag</code>	(numpy array) NumPy array that defines the flag of the border reached with the specified velocity
<code>valin</code>	(int) value in the fluid domain
<code>valout</code>	(int) value in the fluid domain

Methods

<code>check_dictionary(dico)</code>	Check the validity of the dictionary which define the domain.
<code>construct_mpi_topology(dico)</code>	Create the mpi topology
<code>create_coords()</code>	Create the coordinates of the interior domain and the whole domain with halo points.
<code>get_bounds()</code>	Return the coordinates of the bottom right and upper left corner of the interior domain.
<code>get_bounds_halo()</code>	Return the coordinates of the bottom right and upper left corner of the whole domain with halo points.
<code>list_of_labels()</code>	Get the list of all the labels used in the geometry.
<code>visualize([viewer_app, view_distance, ...])</code>	Visualize the domain by creating a plot.

pyLBM.Domain.check_dictionary

`Domain.check_dictionary(dico)`

Check the validity of the dictionary which define the domain.

pyLBM.Domain.construct_mpi_topology

`Domain.construct_mpi_topology(dico)`
Create the mpi topology

pyLBM.Domain.create_coords

`Domain.create_coords()`
Create the coordinates of the interior domain and the whole domain with halo points.

pyLBM.Domain.get_bounds

`Domain.get_bounds()`
Return the coordinates of the bottom right and upper left corner of the interior domain.

pyLBM.Domain.get_bounds_halo

`Domain.get_bounds_halo()`
Return the coordinates of the bottom right and upper left corner of the whole domain with halo points.

pyLBM.Domain.list_of_labels

`Domain.list_of_labels()`
Get the list of all the labels used in the geometry.

pyLBM.Domain.visualize

`Domain.visualize(viewer_app=<module 'pyLBM.viewer.matplotlibViewer' from
'/home/docs/checkouts/readthedocs.org/user_builds/pylbm-
loic/conda/fix-readthedoc/lib/python2.7/site-packages/pyLBM-0.3.0-
py2.7.egg/pyLBM/viewer/matplotlibViewer.pyc'>, view_distance=False,
view_in=True, view_out=True, view_bound=False, label=None)`
Visualize the domain by creating a plot.

Parameters `viewer_app` : Viewer, optional

define the viewer to plot the domain default is viewer.matplotlibViewer

view_distance : boolean or int or list of int, optional

view the distance between the interior points and the border default is False

view_in : boolean, optional

view the inner points default is True

view_out : boolean, optional

view the outer points default is True

view_bound : boolean, optional

view the points on the bounds default is False

label : int or list of int, optional

view the distance only for the specified labels

Returns a figure representing the domain

pyLBM.Scheme

class `pyLBM.Scheme` (*dico, stencil=None*)

Create the class with all the needed informations for each elementary scheme.

Parameters `dico` : a dictionary that contains the following *key:value*

- `dim` : spatial dimension (optional if the *box* is given)
- `scheme_velocity` : the value of the ratio space step over time step ($la = dx / dt$)
- `schemes` : a list of dictionaries, one for each scheme
- `generator` : a generator for the code, optional (see `Generator`)
- `ode_solver` : a method to integrate the source terms, optional (see `ode_solver`)
- `test_stability` : boolean (optional)

Notes

Each dictionary of the list *schemes* should contains the following *key:value*

- `velocities` : list of the velocities number
- `conserved_moments` : list of the moments conserved by each scheme
- `polynomials` : list of the polynomial functions that define the moments
- `equilibrium` : list of the values that define the equilibrium
- `relaxation_parameters` : list of the value of the relaxation parameters
- `source_terms` : dictionary do define the source terms (optional, see examples)
- `init` : dictionary to define the initial conditions (see examples)

If the stencil has already been computed, it can be pass in argument.

Examples

see `demo/examples/scheme/`

Attributes

<code>dim</code>	(int) spatial dimension
<code>dx</code>	(double) space step
<code>dt</code>	(double) time step
<code>la</code>	(double) scheme velocity, ratio dx/dt
<code>nscheme</code>	(int) number of elementary schemes
<code>stencil</code>	(object of class <i>Stencil</i>) a stencil of velocities
<code>P</code>	(list of sympy matrix) list of polynomials that define the moments
<code>EQ</code>	(list of sympy matrix) list of the equilibrium functions
<code>s</code>	(list of list of doubles) relaxation parameters (exemple: <code>s[k][l]</code> is the parameter associated to the <code>l</code> th moment in the <code>k</code> th scheme)
<code>M</code>	(sympy matrix) the symbolic matrix of the moments
<code>Mnum</code>	(numpy array) the numeric matrix of the moments ($m = Mnum F$)
<code>invM</code>	(sympy matrix) the symbolic inverse matrix
<code>invMnum</code>	(numpy array) the numeric inverse matrix ($F = invMnum m$)
<code>generator</code>	(Generator) the used generator (<code>NumpyGenerator</code> , <code>CythonGenerator</code> , ...)
<code>ode_solver</code>	(<code>ode_solver</code> ,) the used ODE solver (<code>explicit_euler</code> , <code>heun</code> , ...)

Methods

<code>compute_amplification_matrix(wave_vector)</code>	compute the amplification matrix of one time step of the scheme
<code>compute_amplification_matrix_relaxation()</code>	compute the amplification matrix of the relaxation.
<code>compute_consistency(dicocons)</code>	compute the consistency of the scheme.
<code>create_moments_matrices()</code>	Create the moments matrices M and M^{-1} used to transform the repartition functions into the moments
<code>equilibrium(mm)</code>	Compute the equilibrium
<code>f2m(ff, mm)</code>	Compute the moments m from the distribution functions f
<code>generate(backend, sorder, valin)</code>	Generate the code by using the appropriated generator
<code>is_L2_stable([Nk])</code>	test the L2 stability of the scheme
<code>is_monotonically_stable()</code>	test the monotonical stability of the scheme.
<code>m2f(mm, ff)</code>	Compute the distribution functions f from the moments m
<code>onestimestep(mm, ff, ff_new, in_or_out, valin)</code>	Compute one time step of the Lattice Boltzmann method
<code>relaxation(m)</code>	The relaxation phase on the moments m
<code>set_boundary_conditions(f, m, bc, interface)</code>	Apply the boundary conditions
<code>set_initialization(scheme)</code>	set the initialization functions for the conserved moments.
<code>set_source_terms(scheme)</code>	set the source terms functions for the conserved moments.
<code>source_term(m[, tn, dt, x, y, z])</code>	The integration of the source term on the moments m
<code>transport(f)</code>	The transport phase on the distribution functions f
<code>vp_amplification_matrix(wave_vector)</code>	compute the eigenvalues of the amplification matrix

pyLBM.Scheme.compute_amplification_matrix

`Scheme.compute_amplification_matrix(wave_vector)`
compute the amplification matrix of one time step of the scheme for the given wave vector.

Returns `amplification_matrix` : numpy array

the matrix of one time step of the scheme in the frame of the distribution functions

Notes

The output matrix corresponds to the linear operator involved in the relaxation phase. If the equilibrium is not a linear combination of the conserved moments, a linearization is done around a given state.

pyLBM.Scheme.compute_amplification_matrix_relaxation

`Scheme.compute_amplification_matrix_relaxation()`
compute the amplification matrix of the relaxation.

Returns `amplification_matrix_relaxation` : numpy array

the matrix of the relaxation in the frame of the distribution functions

Notes

The output matrix corresponds to the linear operator involved in the relaxation phase. If the equilibrium is not a linear combination of the conserved moments, a linearization is done around a given state.

pyLBM.Scheme.compute_consistency

`Scheme.compute_consistency(dicocons)`
compute the consistency of the scheme.

FIX: documentation

pyLBM.Scheme.create_moments_matrices

`Scheme.create_moments_matrices()`
Create the moments matrices M and M^{-1} used to transform the repartition functions into the moments

Three versions of these matrices are computed:

- a sympy version M and $invM$ for each scheme
- a numerical version M_{num} and $invM_{num}$ for each scheme
- a global numerical version $M_{numGlob}$ and $invM_{numGlob}$ for all the schemes

pyLBM.Scheme.equilibrium

`Scheme.equilibrium(mm)`
Compute the equilibrium

pyLBM.Scheme.f2m

`Scheme.f2m(ff, mm)`

Compute the moments m from the distribution functions f

pyLBM.Scheme.generate

`Scheme.generate(backend, sorder, valin)`

Generate the code by using the appropriated generator

Notes

The code can be viewed. If S is the scheme

```
>>> print(S.generator.code)
```

pyLBM.Scheme.is_L2_stable

`Scheme.is_L2_stable(Nk=101)`

test the L2 stability of the scheme

Notes

If the equilibrium is not a linear combination of the conserved moments, a linearization is done around a given state.

The test is performed for Nk^d (default value $Nk=101$) wave vectors uniformly distributed in $[0, 2\pi]^d$ where d is the spatial dimension.

pyLBM.Scheme.is_monotonically_stable

`Scheme.is_monotonically_stable()`

test the monotonical stability of the scheme.

pyLBM.Scheme.m2f

`Scheme.m2f(mm, ff)`

Compute the distribution functions f from the moments m

pyLBM.Scheme.onestimestep

`Scheme.onestimestep(mm, ff, ff_new, in_or_out, valin, tn=0.0, dt=0.0, x=0.0, y=0.0, z=0.0)`

Compute one time step of the Lattice Boltzmann method

pyLBM.Scheme.relaxation

`Scheme.relaxation(m)`

The relaxation phase on the moments *m*

pyLBM.Scheme.set_boundary_conditions

`Scheme.set_boundary_conditions(f, m, bc, interface)`

Apply the boundary conditions

Parameters *f*: numpy array

the array of the distribution functions

m: numpy array

the array of the moments

bc: `pyLBM.boundary.Boundary`

the class that contains all the informations needed for the boundary conditions

Returns Modify the array of the distribution functions *f* in the phantom border area according to the labels. In the direction parallel to the bounday, *N* denotes the number of inner points, phantom cells are added to take into account the boundary conditions.

Notes

If *n* is the number of outer cells on each bound and *N* the number of inner cells, the following representation could be usefull ($N_a = N + 2 * n$)

n outer cells	N inner cells	n outer cells
	0 ... N-1	
0 ... n-1	n ... N+n-1	N+n ... N _a -1

pyLBM.Scheme.set_initialization

`Scheme.set_initialization(scheme)`

set the initialization functions for the conserved moments.

Parameters *scheme*: dictionnary that describes the LBM schemes

Returns *init*: dictionnary where the keys are the indices of the conserved moments and the values must be

- a constant (int or float)
- a tuple of size 2 that describes a function and its extra args

pyLBM.Scheme.set_source_terms

`Scheme.set_source_terms(scheme)`

set the source terms functions for the conserved moments.

Parameters `scheme` : dictionary that describes the LBM schemes

Returns `source_terms` : dictionary where the keys are the indices of the conserved moments
and the values must be a sympy expression or None

pyLBM.Scheme.source_term

`Scheme.source_term(m, tn=0.0, dt=0.0, x=0.0, y=0.0, z=0.0)`
The integration of the source term on the moments m

pyLBM.Scheme.transport

`Scheme.transport(f)`
The transport phase on the distribution functions f

pyLBM.Scheme.vp_amplification_matrix

`Scheme.vp_amplification_matrix(wave_vector)`
compute the eigenvalues of the amplification matrix for a given wave vector.
Returns `eigenvalues` : numpy array
the complex eigenvalues computed by `numpy.linalg.eig`

pyLBM.Simulation

`class pyLBM.Simulation(dico, domain=None, scheme=None, sorder=None, dtype='float64')`
create a class simulation

Parameters `dico` : dictionary
domain : object of class *Domain*, optional
scheme : object of class *Scheme*, optional
type : optional argument (default value is 'float64')

Notes

The methods `transport`, `relaxation`, `equilibrium`, `f2m`, `m2f`, `boundary_condition`, and `one_time_step` are just call of the methods of the class *Scheme*.

Examples

see `demo/examples/`

Access to the distribution functions and the moments.

In 1D:

```
>>>F[n][k][i]
>>>m[n][k][i]
```


get the k th distribution function of the n th elementary scheme and the k th moment of the n th elementary scheme at the point $x[0][i]$.

In 2D:

```
>>>F[n][k][j, i]
>>>m[n][k][j, i]
```

get the k th distribution function of the n th elementary scheme and the k th moment of the n th elementary scheme at the point $x[0][i]$, $x[1][j]$.

Attributes

<i>m</i>	get the moment i in the interior domain.
<i>F</i>	get the distribution function i in the interior domain.
<i>m_halo</i>	get the moment i on the whole domain with halo points.
<i>F_halo</i>	get the distribution function i on the whole domain with halo points.

pyLBM.Simulation.m

`Simulation.m`

get the moment i in the interior domain.

pyLBM.Simulation.F

`Simulation.F`

get the distribution function i in the interior domain.

pyLBM.Simulation.m_halo

`Simulation.m_halo`

get the moment i on the whole domain with halo points.

pyLBM.Simulation.F_halo

`Simulation.F_halo`

get the distribution function i on the whole domain with halo points.

<code>dim</code>	(int) spatial dimension
<code>type</code>	(float64) the type of the values
<code>domain</code>	(<i>Domain</i>) the domain given in argument
<code>scheme</code>	(<i>Scheme</i>) the scheme given in argument

Methods

<code>boundary_condition()</code>	perform the boundary conditions
Continued on next page	

Table 2.7 – continued from previous page

<code>equilibrium()</code>	set the moments to the equilibrium values
<code>f2m()</code>	compute the moments from the distribution functions
<code>initialization(dico)</code>	initialize all the numpy array with the initial conditions
<code>m2f()</code>	compute the distribution functions from the moments
<code>one_time_step()</code>	compute one time step
<code>relaxation()</code>	compute the relaxation phase on moments
<code>source_term([fraction_of_time_step])</code>	compute the source term phase on moments
<code>time_info()</code>	
<code>transport()</code>	compute the transport phase on distribution functions

pyLBM.Simulation.boundary_condition

`Simulation.boundary_condition()`
perform the boundary conditions

Notes

The array `_F` is modified in the phantom array (outer points) according to the specified boundary conditions.

pyLBM.Simulation.equilibrium

`Simulation.equilibrium()`
set the moments to the equilibrium values (the array `_m` is modified)

Notes

Another moments vector can be set to equilibrium values: use directly the method of the class `Scheme`

pyLBM.Simulation.f2m

`Simulation.f2m()`
compute the moments from the distribution functions (the array `_m` is modified)

pyLBM.Simulation.initialization

`Simulation.initialization(dico)`
initialize all the numpy array with the initial conditions

Parameters `dico` : the dictionary with the *key:value* ‘init’

Returns set the initial values to the numpy arrays `_F` and `_m`

Notes

The initial values are set to `_m`, the array `_F` is then initialized with the equilibrium values. If the initial values have to be set to `_F`, use the optional *key:value* ‘inittype’ with the value ‘distributions’ (default value is set to ‘moments’).

pyLBM.Simulation.m2f

`Simulation.m2f()`
 compute the distribution functions from the moments (the array `_F` is modified)

pyLBM.Simulation.one_time_step

`Simulation.one_time_step()`
 compute one time step

Notes

Modify the arrays `_F` and `_m` in order to go further of `dt`. This function is equivalent to successively use

- boundary_condition
- transport
- f2m
- relaxation
- m2f

pyLBM.Simulation.relaxation

`Simulation.relaxation()`
 compute the relaxation phase on moments (the array `_m` is modified)

pyLBM.Simulation.source_term

`Simulation.source_term(fraction_of_time_step=1.0)`
 compute the source term phase on moments (the array `_m` is modified)

pyLBM.Simulation.time_info

`Simulation.time_info()`

pyLBM.Simulation.transport

`Simulation.ttransport()`
 compute the transport phase on distribution functions (the array `_F` is modified)

The modules

the module stencil

`Stencil(dico)`

Create the stencil of velocities used by the scheme(s).

Continued on next page

Table 2.8 – continued from previous page

<code>OneStencil(v, nv, num2index, nv_ptr)</code>	Create a stencil of a LBM scheme.
<code>Velocity([dim, num, vx, vy, vz])</code>	Create a velocity.

pyLBM.stencil.Stencil

class `pyLBM.stencil.Stencil(dico)`

Create the stencil of velocities used by the scheme(s).

The numbering of the velocities follows the convention for 1D and 2D.

Parameters `dico` : a dictionary that contains the following *key:value*

- `dim` : the value of the spatial dimension (1, 2 or 3)
- `schemes` : a list of the dictionaries that contain the *key:value* velocities
`[{'velocities':[...]}, {'velocities':[...]}, {'velocities':[...]}, ...]`

Notes

The velocities for each schemes are defined as a Python list.

Examples

```
>>> s = Stencil({'dim': 1,
                'schemes':[{'velocities': range(9)}, ],
                })
>>> s
Stencil informations
* spatial dimension: 1
* maximal velocity in each direction: [4 None None]
* minimal velocity in each direction: [-4 None None]
* Informations for each elementary stencil:
  stencil 0
    - number of velocities: 9
    - velocities: (0: 0), (1: 1), (2: -1), (3: 2), (4: -2), (5: 3), (6: -3),
  ↪ (7: 4), (8: -4),
```

```
>>> s = Stencil({'dim': 2,
                'schemes':[{'velocities':range(9)},
                           {'velocities':range(50)}],
                })
>>> s
Stencil informations
* spatial dimension: 2
* maximal velocity in each direction: [4 3 None]
* minimal velocity in each direction: [-3 -3 None]
* Informations for each elementary stencil:
  stencil 0
    - number of velocities: 9
    - velocities: (0: 0, 0), (1: 1, 0), (2: 0, 1), (3: -1, 0), (4: 0, -1),
  ↪ (5: 1, 1), (6: -1, 1), (7: -1, -1), (8: 1, -1),
  stencil 1
```

```

- number of velocities: 50
- velocities: (0: 0, 0), (1: 1, 0), (2: 0, 1), (3: -1, 0), (4: 0, -1),
→(5: 1, 1), (6: -1, 1), (7: -1, -1), (8: 1, -1), (9: 2, 0), (10: 0, 2), (11: -2,
→0), (12: 0, -2), (13: 2, 2), (14: -2, 2), (15: -2, -2), (16: 2, -2), (17: 2, 1),
→(18: 1, 2), (19: -1, 2), (20: -2, 1), (21: -2, -1), (22: -1, -2), (23: 1, -2),
→(24: 2, -1), (25: 3, 0), (26: 0, 3), (27: -3, 0), (28: 0, -3), (29: 3, 3), (30:
→-3, 3), (31: -3, -3), (32: 3, -3), (33: 3, 1), (34: 1, 3), (35: -1, 3), (36: -3,
→1), (37: -3, -1), (38: -1, -3), (39: 1, -3), (40: 3, -1), (41: 3, 2), (42: 2,
→3), (43: -2, 3), (44: -3, 2), (45: -3, -2), (46: -2, -3), (47: 2, -3), (48: 3, -
→2), (49: 4, 0),

```

get the x component of the unique velocities

```

>>> s.uvx
array([ 0,  1,  0, -1,  0,  1, -1, -1,  1,  2,  0, -2,  0,  2, -2, -2,  2,
        2,  1, -1, -2, -2, -1,  1,  2,  3,  0, -3,  0,  3, -3, -3,  3,  3,
        1, -1, -3, -3, -1,  1,  3,  3,  2, -2, -3, -3, -2,  2,  3,  4])

```

get the y component of the velocity for the second stencil

```

>>> s.vy[1]
array([ 0,  0,  1,  0, -1,  1,  1, -1, -1,  0,  2,  0, -2,  2,  2, -2, -2,
        1,  2,  2,  1, -1, -2, -2, -1,  0,  3,  0, -3,  3,  3, -3, -3,  1,
        3,  3,  1, -1, -3, -3, -1,  2,  3,  3,  2, -2, -3, -3, -2,  0])

```

Attributes

<code>uvx</code>	the x component of the unique velocities.
<code>uvy</code>	the y component of the unique velocities.
<code>uvz</code>	the z component of the unique velocities.
<code>unum</code>	the numbering of the unique velocities.
<code>vmax</code>	the maximal velocity in norm for each spatial direction.
<code>vmin</code>	the minimal velocity in norm for each spatial direction.
<code>vx</code>	<code>vx[k]</code> the x component of the velocities for the stencil <code>k</code> .
<code>vy</code>	<code>vy[k]</code> the y component of the velocities for the stencil <code>k</code> .
<code>vz</code>	<code>vz[k]</code> the z component of the velocities for the stencil <code>k</code> .
<code>num</code>	<code>num[k]</code> the numbering of the velocities for the stencil <code>k</code> .
<code>unvtot</code>	the number of unique velocities involved in the stencils.

pyLBM.stencil.Stencil.uvx

`Stencil.uvx`

the x component of the unique velocities.

pyLBM.stencil.Stencil.uvy

`Stencil.uvy`

the y component of the unique velocities.

pyLBM.stencil.Stencil.uvz

`Stencil.uvz`
the z component of the unique velocities.

pyLBM.stencil.Stencil.unum

`Stencil.unum`
the numbering of the unique velocities.

pyLBM.stencil.Stencil.vmax

`Stencil.vmax`
the maximal velocity in norm for each spatial direction.

pyLBM.stencil.Stencil.vmin

`Stencil.vmin`
the minimal velocity in norm for each spatial direction.

pyLBM.stencil.Stencil.vx

`Stencil.vx`
`vx[k]` the x component of the velocities for the stencil k.

pyLBM.stencil.Stencil.vy

`Stencil.vy`
`vy[k]` the y component of the velocities for the stencil k.

pyLBM.stencil.Stencil.vz

`Stencil.vz`
`vz[k]` the z component of the velocities for the stencil k.

pyLBM.stencil.Stencil.num

`Stencil.num`
`num[k]` the numbering of the velocities for the stencil k.

pyLBM.stencil.Stencil.unvtot

`Stencil.unvtot`
the number of unique velocities involved in the stencils.

<code>dim</code>	(int) the spatial dimension (1, 2 or 3).
<code>unique_velocities</code>	(NumPy array) array of all velocities involved in the stencils. Each unique velocity appeared only once.
<code>nstencils</code>	(int) the number of elementary stencils.
<code>nv</code>	(list of integers) the number of velocities for each elementary stencil.
<code>v</code>	(list of velocities) list of all the velocities for each elementary stencil.
<code>nv_ptr</code>	(list of integers) used to obtain the list of the velocities involved in a stencil. For instance, the list for the <i>k</i> th stencil is <code>v[nv_ptr[k]:nv_ptr[k+1]]</code>

Methods

<code>append</code>	<code>L.append(object)</code> – append object to end
<code>count(...)</code>	
<code>extend</code>	<code>L.extend(iterable)</code> – extend list by appending elements from the iterable
<code>get_all_velocities()</code>	get all the velocities for all the stencils in one array
<code>get_stencil(k)</code>	
<code>get_symmetric([axis])</code>	get the symmetric velocities.
<code>index((value, [start, ...])</code>	Raises <code>ValueError</code> if the value is not present.
<code>insert</code>	<code>L.insert(index, object)</code> – insert object before index
<code>is_symmetric()</code>	check if all the velocities have their symmetric.
<code>pop(...)</code>	Raises <code>IndexError</code> if list is empty or index is out of range.
<code>remove</code>	<code>L.remove(value)</code> – remove first occurrence of value.
<code>reverse</code>	<code>L.reverse()</code> – reverse <i>IN PLACE</i>
<code>sort</code>	<code>L.sort(cmp=None, key=None, reverse=False)</code> – stable sort <i>IN PLACE</i> ;
<code>visualize([viewer_mod, k, unique_velocities])</code>	plot the velocities

pyLBM.stencil.Stencil.append

`Stencil.append()`
`L.append(object)` – append object to end

pyLBM.stencil.Stencil.count

`Stencil.count(value)` → integer – return number of occurrences of value

pyLBM.stencil.Stencil.extend

`Stencil.extend()`
`L.extend(iterable)` – extend list by appending elements from the iterable

pyLBM.stencil.Stencil.get_all_velocities

`Stencil.get_all_velocities()`
get all the velocities for all the stencils in one array

pyLBM.stencil.Stencil.get_stencil

`Stencil.get_stencil(k)`

pyLBM.stencil.Stencil.get_symmetric

`Stencil.get_symmetric(axis=None)`
get the symetrics velocities.

pyLBM.stencil.Stencil.index

`Stencil.index(value[, start[, stop]])` → integer – return first index of value.
Raises `ValueError` if the value is not present.

pyLBM.stencil.Stencil.insert

`Stencil.insert()`
`L.insert(index, object)` – insert object before index

pyLBM.stencil.Stencil.is_symmetric

`Stencil.is_symmetric()`
check if all the velocities have their symetric.

pyLBM.stencil.Stencil.pop

`Stencil.pop([index])` → item – remove and return item at index (default last).
Raises `IndexError` if list is empty or index is out of range.

pyLBM.stencil.Stencil.remove

`Stencil.remove()`
`L.remove(value)` – remove first occurrence of value. Raises `ValueError` if the value is not present.

pyLBM.stencil.Stencil.reverse

`Stencil.reverse()`
`L.reverse()` – reverse *IN PLACE*

pyLBM.stencil.Stencil.sort

`Stencil.sort()`
`L.sort(cmp=None, key=None, reverse=False)` – stable sort *IN PLACE*; `cmp(x, y)` -> -1, 0, 1

pyLBM.stencil.Stencil.visualize

```
Stencil.visualize (viewer_mod=<module      'pyLBM.viewer.matplotlibViewer'      from
                  '/home/docs/checkouts/readthedocs.org/user_builds/pylbm-
                  loic/conda/fix-readthedoc/lib/python2.7/site-packages/pyLBM-
                  0.3.0-py2.7.egg/pyLBM/viewer/matplotlibViewer.pyc'>,      k=None,
                  unique_velocities=False)
plot the velocities
```

Parameters **viewer** : package used to plot the figure (could be matplotlib, vtk, ...)

see viewer for more information

k : list of stencil index to plot

if None plot all stencils

unique_velocities : if True plot the unique velocities

pyLBM.stencil.OneStencil

class `pyLBM.stencil.OneStencil` (*v*, *nv*, *num2index*, *nv_ptr*)
Create a stencil of a LBM scheme.

Parameters **v** : list

the list of the velocities of that stencil

nv : int

size of the list

num2index : list of integers

link between the velocity number and its position in the unique velocities array

Attributes

<i>num</i>	the numbering of the velocities.
<i>vx</i>	the x component of the velocities.
<i>vy</i>	the y component of the velocities.
<i>vz</i>	the z component of the velocities.

pyLBM.stencil.OneStencil.num

`OneStencil.num`
the numbering of the velocities.

pyLBM.stencil.OneStencil.vx

`OneStencil.vx`
the x component of the velocities.

pyLBM.stencil.OneStencil.vy

`OneStencil.vy`
the y component of the velocities.

pyLBM.stencil.OneStencil.vz

`OneStencil.vz`
the z component of the velocities.

<code>v</code>	(list) the list of the velocities of that stencil
<code>nv</code>	(int) size of the list <code>v</code>
<code>num2index</code>	(list of integers) link between the velocity number and its position in the unique velocities array

pyLBM.stencil.Velocity

class `pyLBM.stencil.Velocity` (*dim=None, num=None, vx=None, vy=None, vz=None*)
Create a velocity.

Parameters `dim` : int, optional

The dimension of the velocity.

`num` : int, optional

The number of the velocity in the numbering convention of Lattice-Boltzmann scheme.

`vx` : int, optional

The x component of the velocity vector.

`vy` : int, optional

The y component of the velocity vector.

`vz` : int, optional

The z component of the velocity vector.

Notes

```
from __future__ import print_function, division
from six.moves import range
import matplotlib
import matplotlib.pyplot as plt
import matplotlib.colors as colors
import matplotlib.cm as cm
from matplotlib.patches import FancyArrowPatch
from mpl_toolkits.mplot3d import Axes3D, proj3d

import pyLBM
import numpy as np

class Arrow3D(FancyArrowPatch):
    def __init__(self, xs, ys, zs, *args, **kwargs):
        FancyArrowPatch.__init__(self, (0,0), (0,0), *args, **kwargs)
```

```

        self._verts3d = xs, ys, zs

    def draw(self, renderer):
        xs3d, ys3d, zs3d = self._verts3d
        xs, ys, zs = proj3d.proj_transform(xs3d, ys3d, zs3d, renderer.M)
        self.set_positions((xs[0],ys[0]),(xs[1],ys[1]))
        FancyArrowPatch.draw(self, renderer)

def Velocities_1D(n):
    dim = 1
    fig = plt.figure(dim, figsize=(8, 4), facecolor='white')
    fig.clf()
    xmin, xmax, ymin, ymax = 1000, -1000, -1, 1
    e = 0.2
    for k in range((2*n+1)**dim):
        v = pyLBM.stencil.Velocity(dim = dim, num = k)
        x = v.vx
        xmin = min(xmin, x)
        xmax = max(xmax, x)
        couleur_texte = 0.
        couleur_trait = 0.5
        plt.text(x, 0., str(v.num), color=[couleur_texte]*3,
                 horizontalalignment='center',verticalalignment='center',
                 fontsize=15)

    plt.plot([xmin, xmax], [0, 0], ':', color=[couleur_trait]*3)
    plt.text(0., ymax+2*e, "Velocities numbering {0:1d}D".format(dim), fontsize=20,
             verticalalignment='center', horizontalalignment='center', color='b')
    plt.arrow(xmin-e, ymin-e, 1, 0, head_width=0.05*dim, head_length=0.1, fc='b', ec='b')
    plt.text(xmin-e+.5, ymin-1.5*e, 'x', color='b',
             verticalalignment='center', horizontalalignment='center')
    plt.axis('off')
    plt.xlim(xmin-2*e, xmax+2*e)
    plt.ylim(ymin-2*e, ymax+2*e)
    plt.draw()

def Velocities_2D(n):
    dim = 2
    fig = plt.figure(dim, figsize=(8, 8), facecolor='white')
    fig.clf()
    xmin, xmax, ymin, ymax = 1000, -1000, 1000, -1000
    e = .5
    for k in range((2*n+1)**dim):
        v = pyLBM.stencil.Velocity(dim = dim, num = k)
        x = v.vx
        y = v.vy
        xmin = min(xmin, x)
        xmax = max(xmax, x)
        ymin = min(ymin, y)
        ymax = max(ymax, y)
        couleur_texte = 0.
        couleur_trait = 0.5
        plt.text(x, y, str(v.num), color=[couleur_texte]*3,
                 horizontalalignment='center',verticalalignment='center',
                 fontsize=15)
    for x in range(xmin, xmax+1):
        plt.plot([x, x], [ymin, ymax], ':', color=[couleur_trait]*3)

```

```

    for y in range(ymin, ymax+1):
        plt.plot([xmin, xmax], [y, y], ':', color=[couleur_trait]*3)
        plt.text(0., ymax+2*e, "Velocities numbering {0:1d}D".format(dim), fontsize=20,
                 verticalalignment='center', horizontalalignment='center', color='b')
        plt.arrow(xmin-e, ymin-e, 1, 0, head_width=0.05*dim, head_length=0.1, fc='b',
                  ec='b')
        plt.arrow(xmin-e, ymin-e, 0, 1, head_width=0.05*dim, head_length=0.1, fc='b',
                  ec='b')
        plt.text(xmin-e+.5, ymin-1.5*e, 'x', color='b',
                 verticalalignment='center', horizontalalignment='center')
        plt.text(xmin-1.5*e, ymin-e+.5, 'y', color='b',
                 verticalalignment='center', horizontalalignment='center')
        plt.axis('off')
        plt.xlim(xmin-2*e, xmax+2*e)
        plt.ylim(ymin-2*e, ymax+2*e)
        plt.draw()

def Velocities_3D(n):
    dim = 3
    couleur_tour = "k"
    fig = plt.figure(dim, figsize=(8, 8), facecolor='white')
    fig.clf()
    ax = fig.add_subplot(111, projection='3d')
    xmin, xmax, ymin, ymax, zmin, zmax = 1000, -1000, 1000, -1000, 1000, -1000
    e = .5
    for k in range((2*n+1)**dim):
        v = pyLBM.stencil.Velocity(dim = dim, num = k)
        x = v.vx
        y = v.vy
        z = v.vz
        xmin = min(xmin, x)
        xmax = max(xmax, x)
        ymin = min(ymin, y)
        ymax = max(ymax, y)
        zmin = min(zmin, z)
        zmax = max(zmax, z)
        couleur_texte = [.5+.5*x, 0., .5-.5*x]
        couleur_trait = 0.5
        ax.text(x, y, z, str(v.num), color=couleur_texte,
                horizontalalignment='center', verticalalignment='center',
                fontsize=15)
    for x in range(xmin, xmax+1):
        for y in range(ymin, ymax+1):
            ax.plot([x, x], [y, y], [zmin, zmax], ':', color=[couleur_trait]*3)
    for x in range(xmin, xmax+1):
        for z in range(zmin, zmax+1):
            ax.plot([x, x], [ymin, ymax], [z, z], ':', color=[couleur_trait]*3)
    for z in range(zmin, zmax+1):
        for y in range(ymin, ymax+1):
            ax.plot([xmin, xmax], [y, y], [z, z], ':', color=[couleur_trait]*3)

    XS, YS = np.meshgrid([-1,1], [-1,1])
    ZS = np.zeros(XS.shape)
    couleur_plan = .8
    for x in [-1,0,1]:
        ax.plot_surface(ZS+x, XS, YS,
                        rstride=1, cstride=1, color=[.5+.5*x, 0., .5-.5*x],
                        shade=False, alpha=0.2,

```

```

        antialiased=False, linewidth=0)
    ax.text(0., 0., zmax+2*e, "Velocities numbering {0:1d}D".format(dim),
    ↪fontsize=20,
        verticalalignment='center', horizontalalignment='center', color=couleur_
    ↪tour)
    vx = Arrow3D([xmax+e,xmax+e+1],[ymin-e,ymin-e],[zmin-e,zmin-e],
        mutation_scale=20, lw=1, arrowstyle="->", color=couleur_tour)
    ax.add_artist(vx)
    vy = Arrow3D([xmax+e,xmax+e],[ymin-e,ymin-e+1],[zmin-e,zmin-e],
        mutation_scale=20, lw=1, arrowstyle="->", color=couleur_tour)
    ax.add_artist(vy)
    vz = Arrow3D([xmax+e,xmax+e],[ymin-e,ymin-e],[zmin-e,zmin-e+1],
        mutation_scale=20, lw=1, arrowstyle="->", color=couleur_tour)
    ax.add_artist(vz)
    ax.text(xmax+e+.8, ymin-.8*e, zmin-e, 'x', color=couleur_tour,
        verticalalignment='center', horizontalalignment='center')
    ax.text(xmax+e, ymin-e+.8, zmin-1.2*e, 'y', color=couleur_tour,
        verticalalignment='center', horizontalalignment='center')
    ax.text(xmax+e, ymin-1.2*e, zmin-e+.8, 'z', color=couleur_tour,
        verticalalignment='center', horizontalalignment='center')
    ax.set_xlim(xmin-e, xmax+e)
    ax.set_ylim(ymin-e, ymax+e)
    ax.set_zlim(zmin-e, zmax+e)
    ax.azim = 34
    ax.elev = 20
    plt.axis('off')
    plt.draw()

def Velocities(dim, n):
    if dim == 1:
        Velocities_1D(n)
    elif dim == 2:
        Velocities_2D(n)
    elif dim == 3:
        Velocities_3D(n)
    else:
        print("error of dimension")
    plt.show()

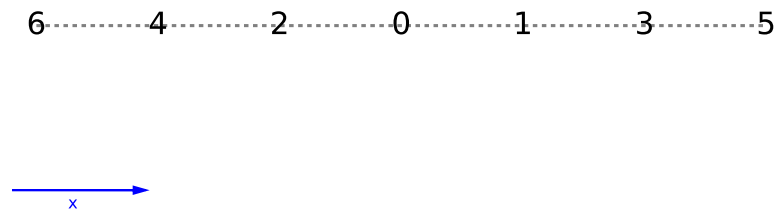
```

```

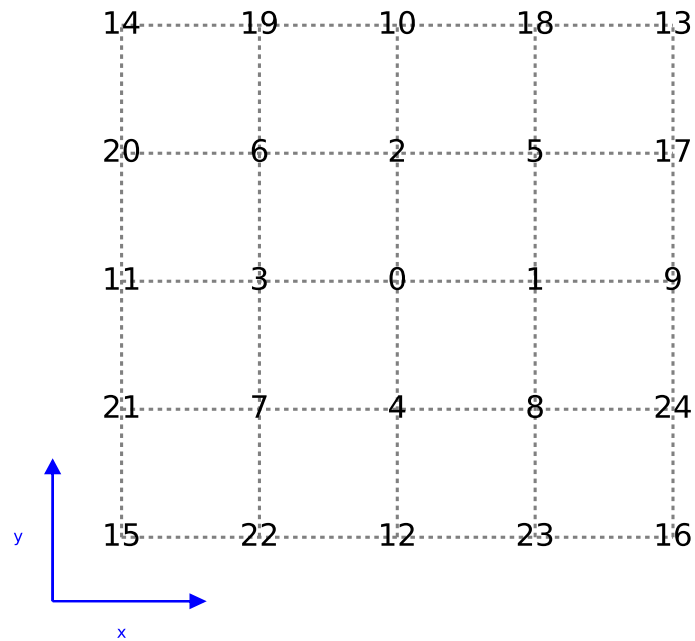
Velocities(1, 3)
Velocities(2, 2)
Velocities(3, 1)

```

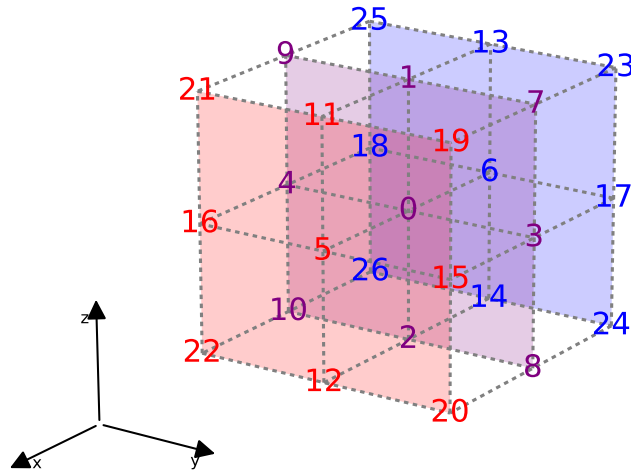
Velocities numbering 1D



Velocities numbering 2D



Velocities numbering 3D



Examples

Create a velocity with the dimension and the number

```
>>> v = Velocity(dim = 1, num = 2)
>>> v
velocity 2
vx: -1
```

Create a velocity with a direction

```
>>> v = Velocity(vx=1, vy=1)
>>> v
velocity 5
vx: 1
vy: 1
```


Attributes

<code>v</code>	velocity
----------------	----------

pyLBM.stencil.Velocity.v

Velocity.**v**
velocity

<code>dim</code>	(int) The dimension of the velocity.
<code>num</code>	The number of the velocity in the numbering convention of Lattice-Boltzmann scheme.
<code>vx</code>	(int) The x component of the velocity vector.
<code>vy</code>	(int) The y component of the velocity vector.
<code>vz</code>	(int) The z component of the velocity vector.

Methods

<code>get_symmetric([axis])</code>	return the symmetric velocity.
<code>set_symmetric()</code>	create the symmetric velocity.

pyLBM.stencil.Velocity.get_symmetric

Velocity.**get_symmetric** (*axis=None*)
return the symmetric velocity.

Parameters `axis` : the axis of the symmetry, optional

(None involves the symmetric with the origin, 0 with the x axis, 1 with the y axis, and 2 with the z axis)

Returns The symmetric of the velocity

pyLBM.stencil.Velocity.set_symmetric

Velocity.**set_symmetric**()
create the symmetric velocity.

The module elements

New in version 0.2: the geometrical elements are yet implemented in 3D.

The module elements contains all the geometrical shapes that can be used to build the geometry.

The 2D elements are:

<code>Circle(center, radius[, label, isfluid])</code>	Class Circle
---	--------------

Continued on next page

Table 2.14 – continued from previous page

<i>Ellipse</i> (center, v1, v2[, label, isfluid])	Class Ellipse
<i>Parallelogram</i> (point, vecta, vectb[, label, ...])	Class Parallelogram
<i>Triangle</i> (point, vecta, vectb[, label, isfluid])	Class Triangle

pyLBM.elements.Circle

class pyLBM.elements.**Circle** (*center, radius, label=0, isfluid=False*)
Class Circle

Parameters **center** : a list that contains the two coordinates of the center

radius : a positive float for the radius

label : list of one integer (default [0])

isfluid : boolean

- True if the circle is added
- False if the circle is deleted

Examples

the circle centered in (0, 0) with radius 1

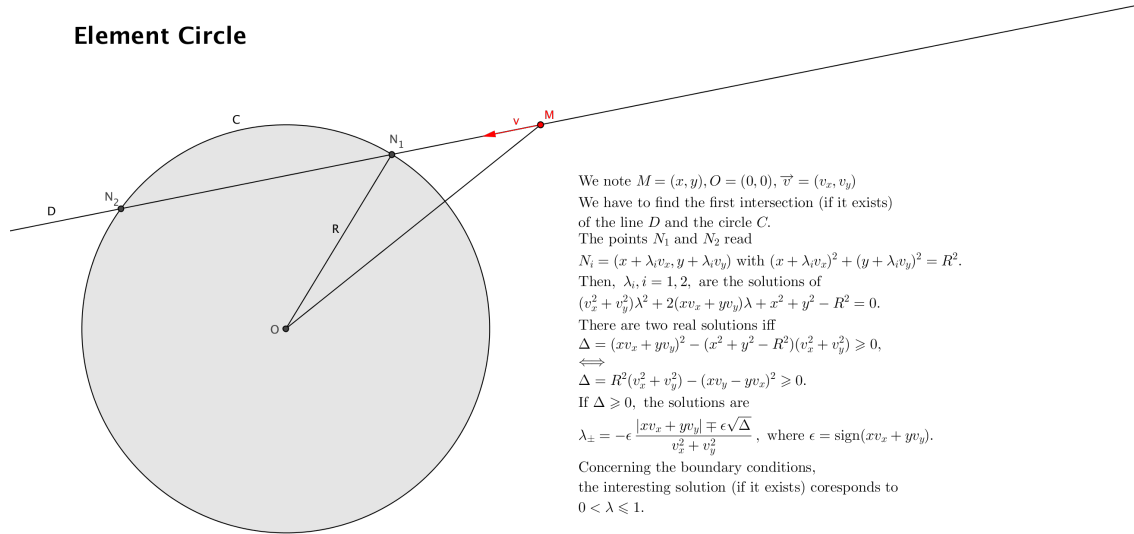
```
>>> center = [0., 0.]
>>> radius = 1.
>>> Circle(center, radius)
Circle([0 0],1) (solid)
```

Attributes

number_of_bounds	(int) 1
center	(numpy array) the coordinates of the center of the circle
radius	(double) positive float for the radius of the circle
label	(list of integers) the list of the label of the edge
isfluid	(boolean) True if the circle is added and False if the circle is deleted

Methods

<i>distance</i> (grid, v[, dmax])	Compute the distance in the v direction between the circle and the points defined by (x, y).
<i>get_bounds</i> ()	Get the bounds of the circle.
<i>point_inside</i> (grid)	return a boolean array which defines
<i>test_label</i> ()	test if the number of labels is equal to the number of bounds.

pyLBM.elements.Circle.distance`Circle.distance (grid, v, dmax=None)`Compute the distance in the v direction between the circle and the points defined by (x, y) .**Element Circle****Parameters** x : x coordinates of the points y : y coordinates of the points v : direction of interest**Returns** array of distances**pyLBM.elements.Circle.get_bounds**`Circle.get_bounds ()`

Get the bounds of the circle.

pyLBM.elements.Circle.point_inside`Circle.point_inside (grid)`

return a boolean array which defines if a point is inside or outside of the circle.

Parameters x : x coordinates of the points y : y coordinates of the points**Returns** Array of boolean (True inside the circle, False otherwise)**Notes**

the edge of the circle is considered as inside.

pyLBM.elements.Circle.test_label

`Circle.test_label()`
test if the number of labels is equal to the number of bounds.

pyLBM.elements.Ellipse

`class pyLBM.elements.Ellipse (center, v1, v2, label=0, isfluid=False)`
Class Ellipse

Parameters **center** : a list that contains the two coordinates of the center

v1 : a vector

v2 : a second vector (v1 and v2 have to be othogonal)

label : list of one integer (default [0])

isfluid : boolean

- True if the ellipse is added
- False if the ellipse is deleted

Examples

the ellipse centered in (0, 0) with v1=[2,0], v2=[0,1]

```
>>> center = [0., 0.]
>>> v1 = [2., 0.]
>>> v2 = [0., 1.]
>>> Ellipse(center, v1, v2)
      Ellipse([0 0], [2 0], [0 1]) (solid)
```

Attributes

number_of_bounds	(int) 1
center	(numpy array) the coordinates of the center of the ellipse
v1	(numpy array) the coordinates of the first vector
v2	(numpy array) the coordinates of the second vector
label	(list of integers) the list of the label of the edge
isfluid	(boolean) True if the ellipse is added and False if the ellipse is deleted
number_of_bounds	(int) number of edges (1)

Methods

<code>distance(grid, v[, dmax])</code>	Compute the distance in the v direction between the ellipse and the points defined by (x, y).
<code>get_bounds()</code>	Get the bounds of the ellipse.
<code>point_inside(grid)</code>	return a boolean array which defines
<code>test_label()</code>	test if the number of labels is equal to the number of bounds.

pyLBM.elements.Ellipse.distance

`Ellipse.distance (grid, v, dmax=None)`

Compute the distance in the v direction between the ellipse and the points defined by (x, y).

Parameters **x** : x coordinates of the points

y : y coordinates of the points

v : direction of interest

Returns array of distances

pyLBM.elements.Ellipse.get_bounds

`Ellipse.get_bounds ()`

Get the bounds of the ellipse.

pyLBM.elements.Ellipse.point_inside

`Ellipse.point_inside (grid)`

return a boolean array which defines if a point is inside or outside of the ellipse.

Parameters **x** : x coordinates of the points

y : y coordinates of the points

Returns Array of boolean (True inside the ellipse, False otherwise)

Notes

the edge of the ellipse is considered as inside.

pyLBM.elements.Ellipse.test_label

`Ellipse.test_label ()`

test if the number of labels is equal to the number of bounds.

pyLBM.elements.Parallelogram

`class pyLBM.elements.Parallelogram (point, vecta, vectb, label=0, isfluid=False)`

Class Parallelogram

Parameters **point** : the coordinates of the first point of the parallelogram

vecta : the coordinates of the first vector

vectb : the coordinates of the second vector

label : list of four integers (default [0, 0, 0, 0])

isfluid : boolean

- True if the parallelogram is added
- False if the parallelogram is deleted

Examples

the square $[0,1] \times [0,1]$

```
>>> point = [0., 0.]
>>> vecta = [1., 0.]
>>> vectb = [0., 1.]
>>> Parallelogram(point, vecta, vectb)
Parallelogram([0 0],[0 1],[1 0]) (solid)
```

Attributes

number_of_bounds	(int) 4
point	(numpy array) the coordinates of the first point of the parallelogram
vecta	(numpy array) the coordinates of the first vector
vectb	(numpy array) the coordinates of the second vector
label	(list of integers) the list of the label of the edge
isfluid	(boolean) True if the parallelogram is added and False if the parallelogram is deleted
number_of_bounds	(int) number of edges (4)

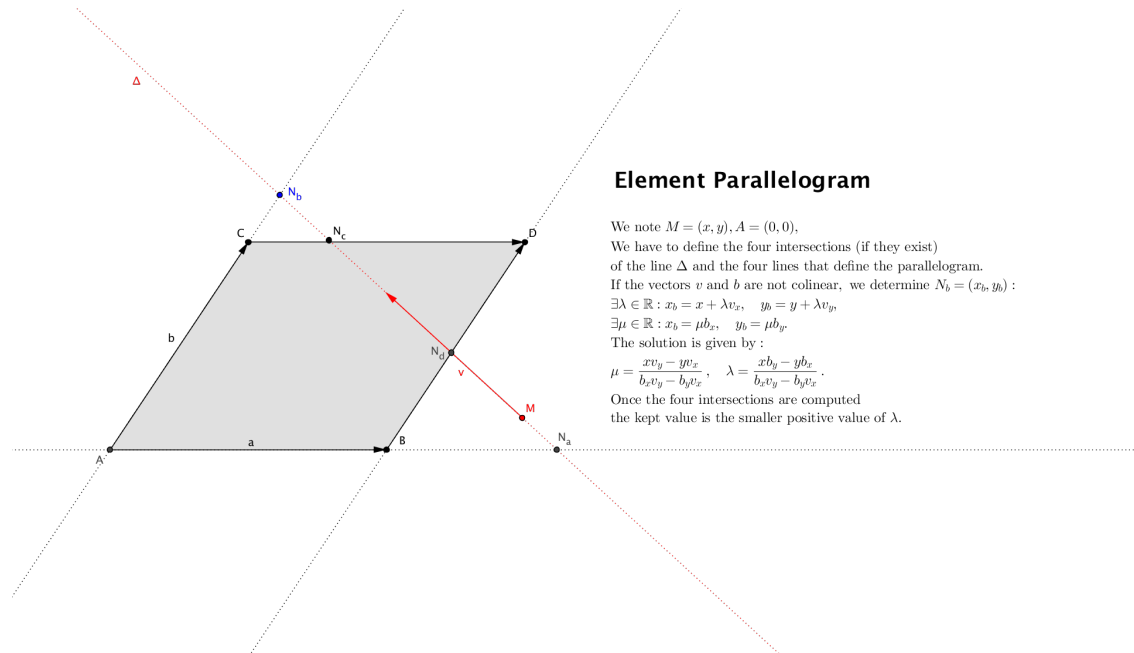
Methods

<i>distance</i> (grid, v[, dmax])	Compute the distance in the v direction between the parallelogram and the points defined by (x, y).
<i>get_bounds</i> ()	return the bounds of the parallelogram.
<i>point_inside</i> (grid)	return a boolean array which defines
<i>test_label</i> ()	test if the number of labels is equal to the number of bounds.

pyLBM.elements.Parallelogram.distance

Parallelogram.**distance** (grid, v, dmax=None)

Compute the distance in the v direction between the parallelogram and the points defined by (x, y).



Parameters **x** : x coordinates of the points

y : y coordinates of the points

v : direction of interest

Returns array of distances

pyLBM.elements.Parallelogram.get_bounds

`Parallelogram.get_bounds()`
 return the bounds of the parallelogram.

pyLBM.elements.Parallelogram.point_inside

`Parallelogram.point_inside(grid)`
 return a boolean array which defines if a point is inside or outside of the parallelogram.

Parameters **x** : x coordinates of the points

y : y coordinates of the points

Returns Array of boolean (True inside the parallelogram, False otherwise)

Notes

the edges of the parallelogram are considered as inside.

pyLBM.elements.Parallelogram.test_label

`Parallelogram.test_label()`
 test if the number of labels is equal to the number of bounds.

pyLBM.elements.Triangle

class pyLBM.elements.**Triangle** (*point, vecta, vectb, label=0, isfluid=False*)
Class Triangle

Parameters **point** : list

the coordinates of the first point of the triangle

vecta : list

the coordinates of the first vector

vectb : list

the coordinates of the second vector

label : list of three integers (default [0, 0, 0])

isfluid : boolean

- True if the triangle is added
- False if the triangle is deleted

Examples

the bottom half square of [0,1]x[0,1]

```
>>> point = [0., 0.]
>>> vecta = [1., 0.]
>>> vectb = [0., 1.]
>>> Triangle(point, vecta, vectb)
Triangle([0 0],[0 1],[1 0]) (solid)
```

Attributes

point	(numpy array) the coordinates of the first point of the triangle
vecta	(numpy array) the coordinates of the first vector
vectb	(numpy array) the coordinates of the second vector
label	(list of integers) the list of the label of the edge
isfluid	(boolean) True if the triangle is added and False if the triangle is deleted
number_of_bounds	(int) number of edges

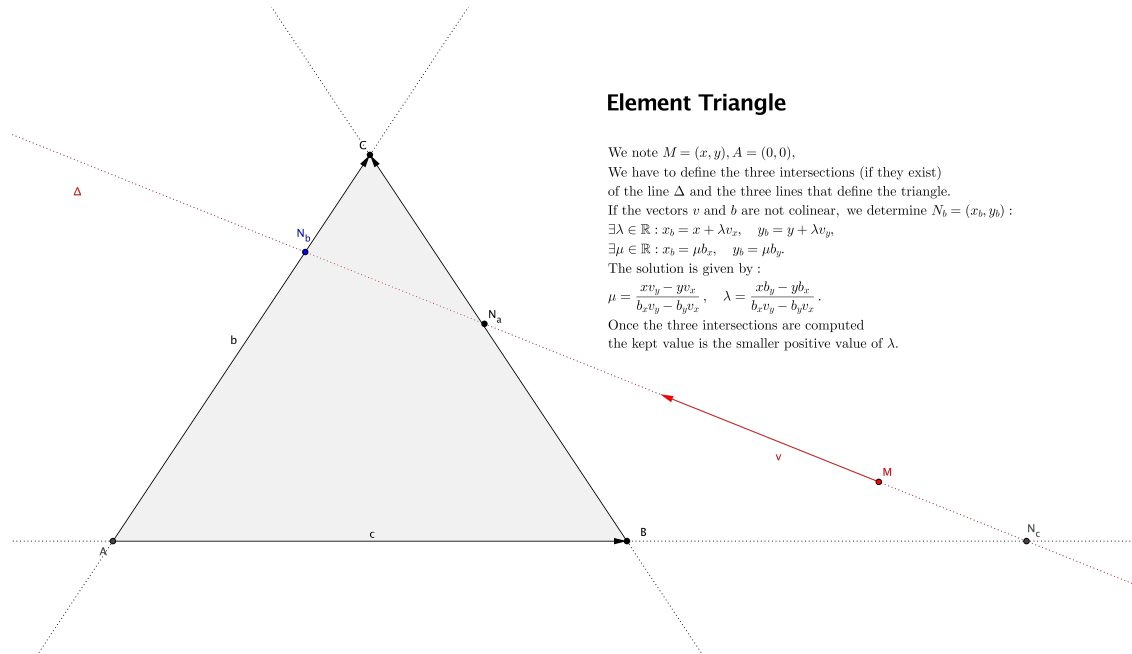
Methods

<code>distance(grid, v[, dmax])</code>	Compute the distance in the v direction between the triangle and the points defined by (x, y).
<code>get_bounds()</code>	return the smallest box where the triangle is.
<code>point_inside(grid)</code>	return a boolean array which defines
<code>test_label()</code>	test if the number of labels is equal to the number of bounds.

pyLBM.elements.Triangle.distance

`Triangle.distance (grid, v, dmax=None)`

Compute the distance in the v direction between the triangle and the points defined by (x, y).



Parameters **x** : x coordinates of the points

y : y coordinates of the points

v : direction of interest

Returns array of distances

pyLBM.elements.Triangle.get_bounds

`Triangle.get_bounds ()`

return the smallest box where the triangle is.

pyLBM.elements.Triangle.point_inside

`Triangle.point_inside (grid)`

return a boolean array which defines if a point is inside or outside of the triangle.

Parameters **x** : x coordinates of the points

y : y coordinates of the points

Returns Array of boolean (True inside the triangle, False otherwise)

Notes

the edges of the triangle are considered as inside.

pyLBM.elements.Triangle.test_label

`Triangle.test_label()`
test if the number of labels is equal to the number of bounds.

The 3D elements are:

<code>Sphere</code> (center, radius[, label, isfluid])	Class Sphere
<code>Ellipsoid</code> (center, v1, v2, v3[, label, isfluid])	Class Ellipsoid
<code>Parallelepiped</code> (point, v0, v1, v2[, label, ...])	Class Parallelepiped
<code>Cylinder_Circle</code> (center, v1, v2, w[, label, ...])	Class Cylinder_Circle
<code>Cylinder_Ellipse</code> (center, v1, v2, w[, label, ...])	Class Cylinder_Ellipse
<code>Cylinder_Triangle</code> (center, v1, v2, w[, ...])	Class Cylinder_Triangle

pyLBM.elements.Sphere

class `pyLBM.elements.Sphere` (*center, radius, label=0, isfluid=False*)
Class Sphere

Parameters **center** : a list that contains the three coordinates of the center

radius : a positive float for the radius

label : list of one integer (default [0])

isfluid : boolean

- True if the sphere is added
- False if the sphere is deleted

Examples

the sphere centered in (0, 0, 0) with radius 1

```
>>> center = [0., 0., 0.]
>>> radius = 1.
>>> Sphere(center, radius)
Sphere([0 0 0],1) (solid)
```

Attributes

<code>number_of_bounds</code>	(int) 1
<code>center</code>	(numpy array) the coordinates of the center of the sphere
<code>radius</code>	(double) positive float for the radius of the sphere
<code>label</code>	(list of integers) the list of the label of the edge
<code>isfluid</code>	(boolean) True if the sphere is added and False if the sphere is deleted
<code>number_of_bounds</code>	(int) number of edges (1)

Methods

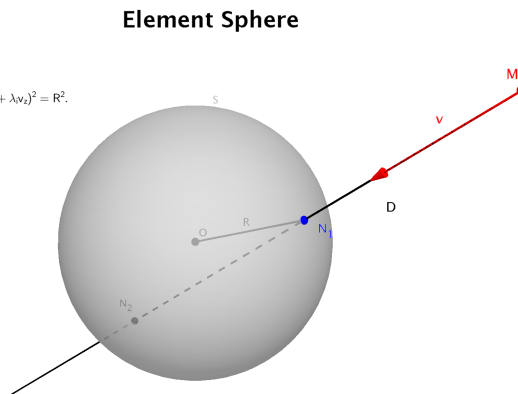
<code>distance(grid, v[, dmax])</code>	Compute the distance in the v direction between the sphere and the points defined by (x, y, z) .
<code>get_bounds()</code>	Get the bounds of the sphere.
<code>point_inside(grid)</code>	return a boolean array which defines
<code>test_label()</code>	test if the number of labels is equal to the number of bounds.

pyLBM.elements.Sphere.distance

`Sphere.distance (grid, v, dmax=None)`

Compute the distance in the v direction between the sphere and the points defined by (x, y, z) .

We note $M = (x, y, z)$, $O = (0, 0, 0)$, $\vec{v} = (v_x, v_y, v_z)$
 We have to find the first intersection (if it exists)
 of the line D and the sphere S .
 The points N_1 and N_2 read
 $N_i = (x + \lambda v_x, y + \lambda v_y, z + \lambda v_z)$ with $(x + \lambda v_x)^2 + (y + \lambda v_y)^2 + (z + \lambda v_z)^2 = R^2$.
 Then, $\lambda_{i=1,2}$ are the solutions of
 $(v_x^2 + v_y^2 + v_z^2)\lambda^2 + 2(xv_x + yv_y + zv_z)\lambda + x^2 + y^2 + z^2 - R^2 = 0$.
 There are two real solutions iff
 $\Delta = (xv_x + yv_y + zv_z)^2 - (x^2 + y^2 + z^2 - R^2)(v_x^2 + v_y^2 + v_z^2) \geq 0$.
 If $\Delta \geq 0$, the solutions are
 $\lambda_{\pm} = -\epsilon \frac{|xv_x + yv_y + zv_z| \mp \epsilon \sqrt{\Delta}}{v_x^2 + v_y^2 + v_z^2}$, where $\epsilon = \text{sign}(xv_x + yv_y + zv_z)$.
 Concerning the boundary conditions,
 the interesting solution (if it exists) corresponds to
 $0 < \lambda \leq 1$.



Parameters x : x coordinates of the points

y : y coordinates of the points

z : z coordinates of the points

v : direction of interest

Returns array of distances

pyLBM.elements.Sphere.get_bounds

`Sphere.get_bounds ()`

Get the bounds of the sphere.

pyLBM.elements.Sphere.point_inside

`Sphere.point_inside (grid)`

return a boolean array which defines if a point is inside or outside of the sphere.

Parameters **x** : x coordinates of the points

y : y coordinates of the points

z : z coordinates of the points

Returns Array of boolean (True inside the sphere, False otherwise)

Notes

the edge of the sphere is considered as inside.

pyLBM.elements.Sphere.test_label

`Sphere.test_label()`

test if the number of labels is equal to the number of bounds.

pyLBM.elements.Ellipsoid

`class pyLBM.elements.Ellipsoid(center, v1, v2, v3, label=0, isfluid=False)`
Class Ellipsoid

Parameters **center** : a list that contains the three coordinates of the center

v1 : a vector

v2 : a vector

v3 : a vector (v1, v2, and v3 have to be orthogonal)

label : list of one integer (default [0])

isfluid : boolean

- True if the ellipsoid is added
- False if the ellipsoid is deleted

Examples

the ellipsoid centered in (0, 0, 0) with v1=[3,0,0], v2=[0,2,0], and v3=[0,0,1]

```
>>> center = [0., 0., 0.]
>>> v1, v2, v3 = [3,0,0], [0,2,0], [0,0,1]
>>> Ellipsoid(center, v1, v2, v3)
Ellipsoid([0 0 0], [3 0 0], [0 2 0], [0 0 1]) (solid)
```

Attributes

number_of_bounds	(int) 1
center	(numpy array) the coordinates of the center of the sphere
v1	(numpy array) the coordinates of the first vector
v2	(numpy array) the coordinates of the second vector
v3	(numpy array) the coordinates of the third vector
label	(list of integers) the list of the label of the edge
isfluid	(boolean) True if the ellipsoid is added and False if the ellipsoid is deleted
number_of_bounds	(int) number of edges (1)

Methods

<code>distance(grid, v[, dmax])</code>	Compute the distance in the v direction between the ellipsoid and the points defined by (x, y, z).
<code>get_bounds()</code>	Get the bounds of the ellipsoid.
<code>point_inside(grid)</code>	return a boolean array which defines
<code>test_label()</code>	test if the number of labels is equal to the number of bounds.

pyLBM.elements.Ellipsoid.distance

`Ellipsoid.distance (grid, v, dmax=None)`

Compute the distance in the v direction between the ellipsoid and the points defined by (x, y, z).

Parameters **x** : x coordinates of the points

y : y coordinates of the points

z : z coordinates of the points

v : direction of interest

Returns array of distances

pyLBM.elements.Ellipsoid.get_bounds

`Ellipsoid.get_bounds ()`

Get the bounds of the ellipsoid.

pyLBM.elements.Ellipsoid.point_inside

`Ellipsoid.point_inside (grid)`

return a boolean array which defines if a point is inside or outside of the ellipsoid.

Parameters **x** : x coordinates of the points

y : y coordinates of the points

z : z coordinates of the points

Returns Array of boolean (True inside the ellipsoid, False otherwise)

Notes

the edge of the ellipsoid is considered as inside.

pyLBM.elements.Ellipsoid.test_label

`Ellipsoid.test_label()`
test if the number of labels is equal to the number of bounds.

pyLBM.elements.Parallelepiped

`class pyLBM.elements.Parallelepiped(point, v0, v1, v2, label=0, isfluid=False)`
Class Parallelepiped

- Parameters**
- point** : a list that contains the three coordinates of the first point
 - v0** : a list of the three coordinates of the first vector that defines the edge
 - v1** : a list of the three coordinates of the second vector that defines the edge
 - v2** : a list of the three coordinates of the third vector that defines the edge
 - label** : list of three integers (default [0,0,0] for the bottom, the top and the side)
 - isfluid** : boolean
 - True if the cylinder is added
 - False if the cylinder is deleted

Examples

the vertical canonical cube centered in (0, 0, 0)

```
>>> center = [0., 0., 0.5]
>>> v0, v1, v2 = [1., 0., 0.], [0., 1., 0.], [0., 0., 1.]
>>> Parallelepiped(center, v0, v1, v2)
Parallelepiped([0 0 0], [1 0 0], [0 1 0], [0 0 1]) (solid)
```

Attributes

number_of_bounds	(int) 6
point	(numpy array) the coordinates of the first point of the parallelepiped
v0	(list of doubles) the three coordinates of the first vector
v1	(list of doubles) the three coordinates of the second vector
v2	(list of doubles) the three coordinates of the third vector
label	(list of integers) the list of the label of the edge
isfluid	(boolean) True if the parallelepiped is added and False if the parallelepiped is deleted
number_of_bounds	(int) number of edges (6)

Methods

<code>change_of_variables()</code>	
<code>distance(grid, v[, dmax])</code>	Compute the distance in the v direction between the cylinder and the points defined by (x, y, z) .
<code>get_bounds()</code>	Get the bounds of the cylinder.
<code>point_inside(grid)</code>	return a boolean array which defines
<code>test_label()</code>	test if the number of labels is equal to the number of bounds.

pyLBM.elements.Parallelepiped.change_of_variables

Parallelepiped.**change_of_variables**()

pyLBM.elements.Parallelepiped.distance

Parallelepiped.**distance**(grid, v, dmax=None)

Compute the distance in the v direction between the cylinder and the points defined by (x, y, z) .

We note $M = (x, y, z)$, $O = (0, 0, 0)$, $\vec{v} = (v_x, v_y, v_z)$ in the frame of the cylinder ($L = 1, R = 1$).

We have to find the first intersection (if it exists) of the line D and the cylinder.

• We first consider the intersections with the side of the cylinder as if it was endless.

The points N_1 and N_2 read

$N_i = (x + \lambda_i v_x, y + \lambda_i v_y, z + \lambda_i v_z)$ with $(x + \lambda_i v_x)^2 + (y + \lambda_i v_y)^2 = R^2$.

Then, $\lambda_i, i = 1, 2$, are the solutions of

$(v_x^2 + v_y^2)\lambda^2 + 2(xv_x + yv_y)\lambda + x^2 + y^2 - R^2 = 0$.

There are two real solutions iff

$\Delta = (xv_x + yv_y)^2 - (x^2 + y^2 - R^2)(v_x^2 + v_y^2) \geq 0$.

If $\Delta \geq 0$, the solutions are

$\lambda_{\pm} = -\epsilon \frac{|xv_x + yv_y| \mp \sqrt{\Delta}}{v_x^2 + v_y^2}$, where $\epsilon = \text{sign}(xv_x + yv_y)$.

The point N_i is on the cylinder if $|z + \lambda_i v_z| \leq 1$.

• We then consider the intersections with the top and the bottom of the cylinder

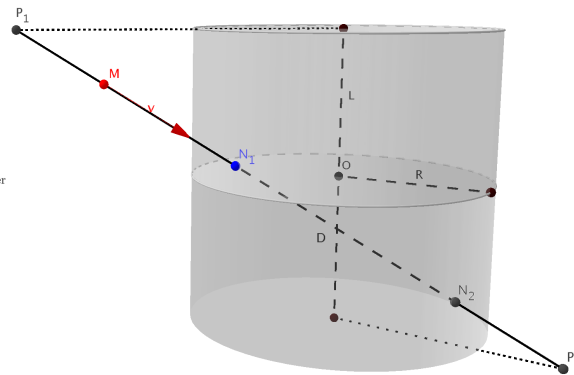
The points P_1 and P_2 read

$P_i = (x + \mu_i v_x, y + \mu_i v_y, z + \mu_i v_z)$ with $z + \mu_1 v_z = 1$ and $z + \mu_2 v_z = -1$.

If $v_z \neq 0$, we have $\mu_1 = (1 - z)/v_z$ and $\mu_2 = -(1 + z)/v_z$.

The point P_i is on the cylinder if $(x + \mu_i v_x)^2 + (y + \mu_i v_y)^2 \leq 1$.

Element Cylinder



Parameters **x** : x coordinates of the points

y : y coordinates of the points

z : z coordinates of the points

v : direction of interest

Returns array of distances

pyLBM.elements.Parallelepiped.get_bounds

Parallelepiped.**get_bounds**()

Get the bounds of the cylinder.

pyLBM.elements.Parallelepiped.point_inside

Parallelepiped.**point_inside** (*grid*)

return a boolean array which defines if a point is inside or outside of the cylinder.

Parameters **x** : x coordinates of the points

y : y coordinates of the points

z : z coordinates of the points

Returns Array of boolean (True inside the cylinder, False otherwise)

Notes

the edge of the cylinder is considered as inside.

pyLBM.elements.Parallelepiped.test_label

Parallelepiped.**test_label** ()

test if the number of labels is equal to the number of bounds.

pyLBM.elements.Cylinder_Circle

class pyLBM.elements.**Cylinder_Circle** (*center, v1, v2, w, label=0, isfluid=False*)

Class Cylinder_Circle

Parameters **center** : a list that contains the three coordinates of the center

v0 : a list of the three coordinates of the first vector that defines the circular section

v1 : a list of the three coordinates of the second vector that defines the circular section

w : a list of the three coordinates of the vector that defines the direction of the side

label : list of three integers (default [0,0,0] for the bottom, the top and the side)

isfluid : boolean

- True if the cylinder is added
- False if the cylinder is deleted

Examples

the vertical canonical cylinder centered in (0, 0, 1/2) with radius 1

```
>>> center = [0., 0., 0.5]
>>> v0, v1 = [1., 0., 0.], [0., 1., 0.]
>>> w = [0., 0., 1.]
>>> Cylinder_Circle(center, v0, v1, w)
Cylinder_Circle([0 0 0.5], [1 0 0], [0 1 0], [0 0 1]) (solid)
```


Attributes

num- ber_of_bounds	(int) 3
center	(numpy array) the coordinates of the center of the cylinder
v0	(list of doubles) the three coordinates of the first vector that defines the base section
v1	(list of doubles) the three coordinates of the second vector that defines the base section
w	(list of doubles) the three coordinates of the vector that defines the direction of the side
label	(list of integers) the list of the label of the edge
isfluid	(boolean) True if the cylinder is added and False if the cylinder is deleted
num- ber_of_bounds	(int) number of edges (3)

Methods

<i>change_of_variables()</i>	
<i>distance</i> (grid, v[, dmax])	Compute the distance in the v direction between the cylinder and the points defined by (x, y, z).
<i>get_bounds()</i>	Get the bounds of the cylinder.
<i>point_inside</i> (grid)	return a boolean array which defines
<i>test_label()</i>	test if the number of labels is equal to the number of bounds.

pyLBM.elements.Cylinder_Circle.change_of_variables

Cylinder_Circle.**change_of_variables**()

pyLBM.elements.Cylinder_Circle.distance

Cylinder_Circle.**distance**(grid, v, dmax=None)

Compute the distance in the v direction between the cylinder and the points defined by (x, y, z).

We note $M = (x, y, z)$, $O = (0, 0, 0)$, $\vec{v} = (v_x, v_y, v_z)$ in the frame of the cylinder ($L = 1, R = 1$).

We have to find the first intersection (if it exists) of the line D and the cylinder.

- We first consider the intersections with the side of the cylinder as if it was endless.

The points N_1 and N_2 read

$$N_i = (x + \lambda_i v_x, y + \lambda_i v_y, z + \lambda_i v_z) \text{ with } (x + \lambda_i v_x)^2 + (y + \lambda_i v_y)^2 = R^2.$$

Then, $\lambda_i, i = 1, 2$, are the solutions of

$$(v_x^2 + v_y^2)\lambda^2 + 2(xv_x + yv_y)\lambda + x^2 + y^2 - R^2 = 0.$$

There are two real solutions iff

$$\Delta = (xv_x + yv_y)^2 - (x^2 + y^2 - R^2)(v_x^2 + v_y^2) \geq 0.$$

If $\Delta \geq 0$, the solutions are

$$\lambda_{\pm} = -\epsilon \frac{|xv_x + yv_y| \mp \sqrt{\Delta}}{v_x^2 + v_y^2}, \text{ where } \epsilon = \text{sign}(xv_x + yv_y).$$

The point N_i is on the cylinder if $|z + \lambda_i v_z| \leq 1$.

- We then consider the intersections with the top and the bottom of the cylinder

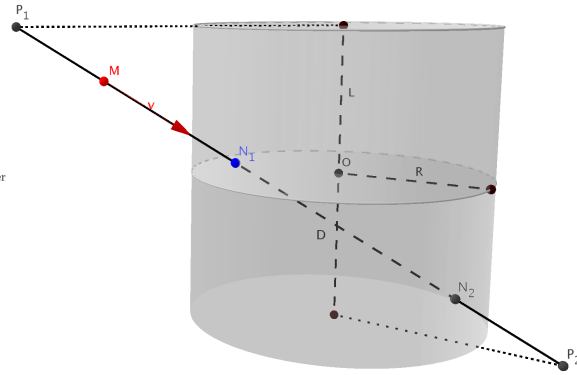
The points P_1 and P_2 read

$$P_i = (x + \mu_i v_x, y + \mu_i v_y, z + \mu_i v_z) \text{ with } z + \mu_1 v_z = 1 \text{ and } z + \mu_2 v_z = -1.$$

If $v_z \neq 0$, we have $\mu_1 = (1 - z)/v_z$ and $\mu_2 = -(1 + z)/v_z$.

The point P_i is on the cylinder if $(x + \mu_i v_x)^2 + (y + \mu_i v_y)^2 \leq 1$.

Element Cylinder



Parameters **x** : x coordinates of the points

y : y coordinates of the points

z : z coordinates of the points

v : direction of interest

Returns array of distances

pyLBM.elements.Cylinder_Circle.get_bounds

`Cylinder_Circle.get_bounds()`

Get the bounds of the cylinder.

pyLBM.elements.Cylinder_Circle.point_inside

`Cylinder_Circle.point_inside(grid)`

return a boolean array which defines if a point is inside or outside of the cylinder.

Parameters **x** : x coordinates of the points

y : y coordinates of the points

z : z coordinates of the points

Returns Array of boolean (True inside the cylinder, False otherwise)

Notes

the edge of the cylinder is considered as inside.

pyLBM.elements.Cylinder_Circle.test_label

`Cylinder_Circle.test_label()`
 test if the number of labels is equal to the number of bounds.

pyLBM.elements.Cylinder_Ellipse

class `pyLBM.elements.Cylinder_Ellipse` (*center, v1, v2, w, label=0, isfluid=False*)
 Class `Cylinder_Ellipse`

Parameters **center** : a list that contains the three coordinates of the center

v0 : a list of the three coordinates of the first vector that defines the circular section

v1 : a list of the three coordinates of the second vector that defines the circular section

w : a list of the three coordinates of the vector that defines the direction of the side

label : list of three integers (default [0,0,0] for the bottom, the top and the side)

isfluid : boolean

- True if the cylinder is added
- False if the cylinder is deleted

Warning: The vectors `v1` and `v2` have to be orthogonal.

Examples

the vertical canonical cylinder centered in (0, 0, 1/2) with radius 1

```
>>> center = [0., 0., 0.5]
>>> v0, v1 = [1., 0., 0.], [0., 1., 0.]
>>> w = [0., 0., 1.]
>>> Cylinder_Ellipse(center, v0, v1, w)
Cylinder_Ellipse([0 0 0.5], [1 0 0], [0 1 0], [0 0 1]) (solid)
```

Attributes

<code>num-ber_of_bounds</code>	(int) 3
<code>center</code>	(numpy array) the coordinates of the center of the cylinder
<code>v0</code>	(list of doubles) the three coordinates of the first vector that defines the base section
<code>v1</code>	(list of doubles) the three coordinates of the second vector that defines the base section
<code>w</code>	(list of doubles) the three coordinates of the vector that defines the direction of the side
<code>label</code>	(list of integers) the list of the label of the edge
<code>isfluid</code>	(boolean) True if the cylinder is added and False if the cylinder is deleted
<code>num-ber_of_bounds</code>	(int) number of edges (3)
<code>num-ber_of_bounds</code>	(int) number of edges (3)

Methods

<code>change_of_variables()</code>	
<code>distance(grid, v[, dmax])</code>	Compute the distance in the v direction between the cylinder and the points defined by (x, y, z).
<code>get_bounds()</code>	Get the bounds of the cylinder.
<code>point_inside(grid)</code>	return a boolean array which defines
<code>test_label()</code>	test if the number of labels is equal to the number of bounds.

pyLBM.elements.Cylinder_Ellipse.change_of_variables

`Cylinder_Ellipse.change_of_variables()`

pyLBM.elements.Cylinder_Ellipse.distance

`Cylinder_Ellipse.distance(grid, v, dmax=None)`

Compute the distance in the v direction between the cylinder and the points defined by (x, y, z).

We note $M = (x, y, z)$, $O = (0, 0, 0)$, $\vec{v} = (v_x, v_y, v_z)$ in the frame of the cylinder ($L = 1, R = 1$).

We have to find the first intersection (if it exists) of the line D and the cylinder.

• We first consider the intersections with the side of the cylinder as if it was endless.

The points N_1 and N_2 read

$N_i = (x + \lambda_i v_x, y + \lambda_i v_y, z + \lambda_i v_z)$ with $(x + \lambda_i v_x)^2 + (y + \lambda_i v_y)^2 = R^2$.

Then, $\lambda_i, i = 1, 2$, are the solutions of

$(v_x^2 + v_y^2)\lambda^2 + 2(xv_x + yv_y)\lambda + x^2 + y^2 - R^2 = 0$.

There are two real solutions iff

$\Delta = (xv_x + yv_y)^2 - (x^2 + y^2 - R^2)(v_x^2 + v_y^2) \geq 0$.

If $\Delta \geq 0$, the solutions are

$\lambda_{\pm} = -\epsilon \frac{xv_x + yv_y \pm \sqrt{\Delta}}{v_x^2 + v_y^2}$, where $\epsilon = \text{sign}(xv_x + yv_y)$.

The point N_i is on the cylinder if $|z + \lambda_i v_z| \leq 1$.

• We then consider the intersections with the top and the bottom of the cylinder

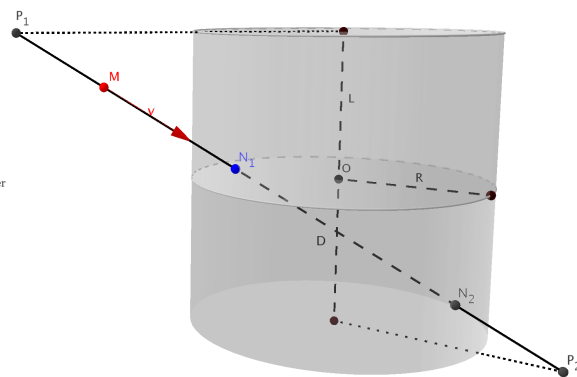
The points P_1 and P_2 read

$P_i = (x + \mu_i v_x, y + \mu_i v_y, z + \mu_i v_z)$ with $z + \mu_1 v_z = 1$ and $z + \mu_2 v_z = -1$.

If $v_z \neq 0$, we have $\mu_1 = (1 - z)/v_z$ and $\mu_2 = -(1 + z)/v_z$.

The point P_i is on the cylinder if $(x + \mu_i v_x)^2 + (y + \mu_i v_y)^2 \leq 1$.

Element Cylinder



Parameters **x** : x coordinates of the points

y : y coordinates of the points

z : z coordinates of the points

v : direction of interest

Returns array of distances

pyLBM.elements.Cylinder_Ellipse.get_bounds

`Cylinder_Ellipse.get_bounds()`

Get the bounds of the cylinder.

pyLBM.elements.Cylinder_Ellipse.point_inside`Cylinder_Ellipse.point_inside(grid)`

return a boolean array which defines if a point is inside or outside of the cylinder.

Parameters **x** : x coordinates of the points**y** : y coordinates of the points**z** : z coordinates of the points**Returns** Array of boolean (True inside the cylinder, False otherwise)**Notes**

the edge of the cylinder is considered as inside.

pyLBM.elements.Cylinder_Ellipse.test_label`Cylinder_Ellipse.test_label()`

test if the number of labels is equal to the number of bounds.

pyLBM.elements.Cylinder_Triangle**class** `pyLBM.elements.Cylinder_Triangle(center, v1, v2, w, label=0, isfluid=False)`

Class Cylinder_Triangle

Parameters **center** : a list that contains the three coordinates of the center**v0** : a list of the three coordinates of the first vector that defines the triangular section**v1** : a list of the three coordinates of the second vector that defines the triangular section**w** : a list of the three coordinates of the vector that defines the direction of the side**label** : list of three integers (default [0,0,0] for the bottom, the top and the side)**isfluid** : boolean

- True if the cylinder is added
- False if the cylinder is deleted

Examples

the vertical canonical cylinder centered in (0, 0, 1/2)

```
>>> center = [0., 0., 0.5]
>>> v0, v1 = [1., 0., 0.], [0., 1., 0.]
>>> w = [0., 0., 1.]
>>> Cylinder_Triangle(center, v0, v1, w)
Cylinder_Triangle([0 0 0.5], [1 0 0], [0 1 0], [0 0 1]) (solid)
```

Attributes

num- ber_of_bounds	(int) 5
center	(numpy array) the coordinates of the center of the cylinder
v0	(list of doubles) the three coordinates of the first vector that defines the base section
v1	(list of doubles) the three coordinates of the second vector that defines the base section
w	(list of doubles) the three coordinates of the vector that defines the direction of the side
label	(list of integers) the list of the label of the edge
isfluid	(boolean) True if the cylinder is added and False if the cylinder is deleted
num- ber_of_bounds	(int) number of edges (3)

Methods

<i>change_of_variables()</i>	
<i>distance</i> (grid, v[, dmax])	Compute the distance in the v direction between the cylinder and the points defined by (x, y, z).
<i>get_bounds()</i>	Get the bounds of the cylinder.
<i>point_inside</i> (grid)	return a boolean array which defines
<i>test_label()</i>	test if the number of labels is equal to the number of bounds.

pyLBM.elements.Cylinder_Triangle.change_of_variables

Cylinder_Triangle.**change_of_variables**()

pyLBM.elements.Cylinder_Triangle.distance

Cylinder_Triangle.**distance**(grid, v, dmax=None)

Compute the distance in the v direction between the cylinder and the points defined by (x, y, z).

We note $M = (x, y, z)$, $O = (0, 0, 0)$, $\vec{v} = (v_x, v_y, v_z)$ in the frame of the cylinder ($L = 1, R = 1$).

We have to find the first intersection (if it exists) of the line D and the cylinder.

• We first consider the intersections with the side of the cylinder as if it was endless.

The points N_1 and N_2 read

$N_i = (x + \lambda_i v_x, y + \lambda_i v_y, z + \lambda_i v_z)$ with $(x + \lambda_i v_x)^2 + (y + \lambda_i v_y)^2 = R^2$.

Then, $\lambda_i, i = 1, 2$, are the solutions of $(v_x^2 + v_y^2)\lambda^2 + 2(xv_x + yv_y)\lambda + x^2 + y^2 - R^2 = 0$.

There are two real solutions iff

$\Delta = (xv_x + yv_y)^2 - (x^2 + y^2 - R^2)(v_x^2 + v_y^2) \geq 0$.

If $\Delta \geq 0$, the solutions are

$\lambda_{\pm} = -\epsilon \frac{|xv_x + yv_y| \mp \sqrt{\Delta}}{v_x^2 + v_y^2}$, where $\epsilon = \text{sign}(xv_x + yv_y)$.

The point N_i is on the cylinder if $|z + \lambda_i v_z| \leq 1$.

• We then consider the intersections with the top and the bottom of the cylinder

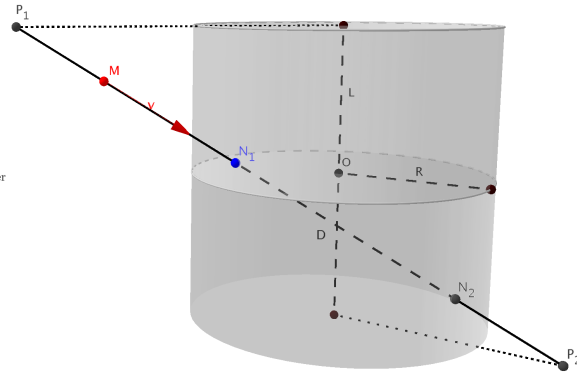
The points P_1 and P_2 read

$P_i = (x + \mu_i v_x, y + \mu_i v_y, z + \mu_i v_z)$ with $z + \mu_1 v_z = 1$ and $z + \mu_2 v_z = -1$.

If $v_z \neq 0$, we have $\mu_1 = (1 - z)/v_z$ and $\mu_2 = -(1 + z)/v_z$.

The point P_i is on the cylinder if $(x + \mu_i v_x)^2 + (y + \mu_i v_y)^2 \leq 1$.

Element Cylinder



Parameters **x** : x coordinates of the points

y : y coordinates of the points

z : z coordinates of the points

v : direction of interest

Returns array of distances

pyLBM.elements.Cylinder_Triangle.get_bounds

`Cylinder_Triangle.get_bounds()`

Get the bounds of the cylinder.

pyLBM.elements.Cylinder_Triangle.point_inside

`Cylinder_Triangle.point_inside(grid)`

return a boolean array which defines if a point is inside or outside of the cylinder.

Parameters **x** : x coordinates of the points

y : y coordinates of the points

z : z coordinates of the points

Returns Array of boolean (True inside the cylinder, False otherwise)

Notes

the edge of the cylinder is considered as inside.

pyLBM.elements.Cylinder_Triangle.test_label

`Cylinder_Triangle.test_label()`
test if the number of labels is equal to the number of bounds.

the module geometry

<code>Geometry(dico)</code>	Create a geometry that defines the fluid part and the solid part.
-----------------------------	---

pyLBM.geometry.Geometry

class `pyLBM.geometry.Geometry(dico)`
Create a geometry that defines the fluid part and the solid part.

Parameters `dico` : a dictionary that contains the following *key:value*

- `box` : a dictionary for the definition of the computed box
- `elements` : a list of elements (optional)

Notes

The dictionary that defines the box should contains the following *key:value*

- `x` : a list of the bounds in the first direction
- `y` : a list of the bounds in the second direction (optional)
- `z` : a list of the bounds in the third direction (optional)
- `label` : an integer or a list of integers (length twice the number of dimensions) used to label each edge (optional)

Examples

see `demo/examples/geometry/`

Attributes

<code>dim</code>	(int) number of spatial dimensions (1, 2, or 3)
<code>bounds</code>	(numpy array) the bounds of the box in each spatial direction
<code>box_label</code>	(list of integers) a list of the four labels for the left, right, bottom, top, front, and back edges
<code>list_elem</code>	(list of elements) a list that contains each element added or deleted in the box

Methods

<code>add_elem(elem)</code>	add a solid or a fluid part in the domain
-----------------------------	---

Continued on next page

Table 2.27 – continued from previous page

<code>list_of_elements_labels()</code>	Get the list of all the labels used in the geometry.
<code>list_of_labels()</code>	Get the list of all the labels used in the geometry.
<code>visualize([viewer_app, viewlabel, fluid_color])</code>	plot a view of the geometry

pyLBM.geometry.Geometry.add_elem

`Geometry.add_elem(elem)`

add a solid or a fluid part in the domain

Parameters `elem` : a geometric element to add (or to del)

pyLBM.geometry.Geometry.list_of_elements_labels

`Geometry.list_of_elements_labels()`

Get the list of all the labels used in the geometry.

pyLBM.geometry.Geometry.list_of_labels

`Geometry.list_of_labels()`

Get the list of all the labels used in the geometry.

pyLBM.geometry.Geometry.visualize

`Geometry.visualize(viewer_app=<module 'pyLBM.viewer.matplotlibViewer' from
'/home/docs/checkouts/readthedocs.org/user_builds/pylbm-
loic/conda/fix-readthedoc/lib/python2.7/site-packages/pyLBM-0.3.0-
py2.7.egg/pyLBM/viewer/matplotlibViewer.pyc'>, viewlabel=False,
fluid_color='blue')`

plot a view of the geometry

Parameters `viewer_app` : a viewer (default matplotlibViewer)

viewlabel : boolean to activate the labels mark (default False)

fluid_color : color for the fluid part (default blue)

the module domain

<code>Domain([dico, geometry, stencil, ...])</code>	Create a domain that defines the fluid part and the solid part and computes the distances between these two states.
---	---

pyLBM.domain.Domain

`class pyLBM.domain.Domain(dico=None, geometry=None, stencil=None, space_step=None, verif=True)`

Create a domain that defines the fluid part and the solid part and computes the distances between these two states.

Parameters `dico` : a dictionary that contains the following *key:value*

- `box` : a dictionary that defines the computational box
- `elements` : the list of the elements (available elements are given in the module `elements`)
- `space_step` : the spatial step
- `schemes` : a list of dictionaries, each of them defining a elementary `Scheme`

Warning: the sizes of the box must be a multiple of the space step `dx`

Notes

The dictionary that defines the box should contains the following *key:value*

- `x` : a list of the bounds in the first direction
- `y` : a list of the bounds in the second direction (optional)
- `z` : a list of the bounds in the third direction (optional)
- `label` : an integer or a list of integers (length twice the number of dimensions) used to label each edge (optional)

See [Geometry](#) for more details.

If the geometry and/or the stencil were previously generated, it can be used directly as following

```
>>> Domain(dico, geometry = geom, stencil = sten)
```

where `geom` is an object of the class [Geometry](#) and `sten` an object of the class [Stencil](#) In that case, `dico` does not need to contain the informations for generate the geometry and/or the stencil

In 1D, `distance[q, i]` is the distance between the point `x[i]` and the border in the direction of the `q`th velocity.

In 2D, `distance[q, j, i]` is the distance between the point `(x[i], y[j])` and the border in the direction of `q`th velocity

In 3D, `distance[q, k, j, i]` is the distance between the point `(x[i], y[j], z[k])` and the border in the direction of `q`th velocity

In 1D, `flag[q, i]` is the flag of the border reached by the point `x[i]` in the direction of the `q`th velocity

In 2D, `flag[q, j, i]` is the flag of the border reached by the point `(x[i], y[j])` in the direction of `q`th velocity

In 3D, `flag[q, k, j, i]` is the flag of the border reached by the point `(x[i], y[j], z[k])` in the direction of `q`th velocity

Examples

see `demo/examples/domain/`

Attributes

<code>x</code>	x component of the coordinates in the interior domain.
<code>y</code>	y component of the coordinates in the interior domain.
<code>z</code>	z component of the coordinates in the interior domain.
Continued on next page	

Table 2.29 – continued from previous page

<code>x_halo</code>	x component of the coordinates of the whole domain (halo points included).
<code>y_halo</code>	y component of the coordinates of the whole domain (halo points included).
<code>z_halo</code>	z component of the coordinates of the whole domain (halo points included).
<code>shape_halo</code>	shape of the whole domain with the halo points.
<code>shape_in</code>	shape of the interior domain.

pyLBM.domain.Domain.x`Domain.x`

x component of the coordinates in the interior domain.

pyLBM.domain.Domain.y`Domain.y`

y component of the coordinates in the interior domain.

pyLBM.domain.Domain.z`Domain.z`

z component of the coordinates in the interior domain.

pyLBM.domain.Domain.x_halo`Domain.x_halo`

x component of the coordinates of the whole domain (halo points included).

pyLBM.domain.Domain.y_halo`Domain.y_halo`

y component of the coordinates of the whole domain (halo points included).

pyLBM.domain.Domain.z_halo`Domain.z_halo`

z component of the coordinates of the whole domain (halo points included).

pyLBM.domain.Domain.shape_halo`Domain.shape_halo`

shape of the whole domain with the halo points.

pyLBM.domain.Domain.shape_in**Domain.shape_in**

shape of the interior domain.

dim	(int) number of spatial dimensions (example: 1, 2, or 3)
global-bounds	(numpy array) the bounds of the box in each spatial direction
bounds	(numpy array) the local bounds of the process in each spatial direction
dx	(double) space step (example: 0.1, 1.e-3)
type	(string) type of data (example: 'float64')
stencil	the stencil of the velocities (object of the class <i>Stencil</i>)
:	
global_size	(list of int) number of points in each direction
extent	(list of int) number of points to add on each side (max velocities)
coords	(numpy array) coordinates of the domain
in_or_out	(numpy array) defines the fluid and the solid part (fluid: value=valin, solid: value=valout)
dis- tance	(numpy array) defines the distances to the borders. The distance is scaled by dx and is not equal to valin only for the points that reach the border with the specified velocity.
flag	(numpy array) NumPy array that defines the flag of the border reached with the specified velocity
valin	(int) value in the fluid domain
valout	(int) value in the fluid domain

Methods

<i>check_dictionary</i> (dico)	Check the validity of the dictionary which define the domain.
<i>construct_mpi_topology</i> (dico)	Create the mpi topology
<i>create_coords</i> ()	Create the coordinates of the interior domain and the whole domain with halo points.
<i>get_bounds</i> ()	Return the coordinates of the bottom right and upper left corner of the interior domain.
<i>get_bounds_halo</i> ()	Return the coordinates of the bottom right and upper left corner of the whole domain with halo points.
<i>list_of_labels</i> ()	Get the list of all the labels used in the geometry.
<i>visualize</i> ([viewer_app, view_distance, ...])	Visualize the domain by creating a plot.

pyLBM.domain.Domain.check_dictionary**Domain.check_dictionary** (*dico*)

Check the validity of the dictionary which define the domain.

pyLBM.domain.Domain.construct_mpi_topology**Domain.construct_mpi_topology** (*dico*)

Create the mpi topology

pyLBM.domain.Domain.create_coords`Domain.create_coords()`

Create the coordinates of the interior domain and the whole domain with halo points.

pyLBM.domain.Domain.get_bounds`Domain.get_bounds()`

Return the coordinates of the bottom right and upper left corner of the interior domain.

pyLBM.domain.Domain.get_bounds_halo`Domain.get_bounds_halo()`

Return the coordinates of the bottom right and upper left corner of the whole domain with halo points.

pyLBM.domain.Domain.list_of_labels`Domain.list_of_labels()`

Get the list of all the labels used in the geometry.

pyLBM.domain.Domain.visualize

```
Domain.visualize (viewer_app=<module      'pyLBM.viewer.matplotlibViewer'      from
                    '/home/docs/checkouts/readthedocs.org/user_builds/pylbm-
                    loic/conda/fix-readthedoc/lib/python2.7/site-packages/pyLBM-0.3.0-
                    py2.7.egg/pyLBM/viewer/matplotlibViewer.pyc'>,      view_distance=False,
                    view_in=True, view_out=True, view_bound=False, label=None)
```

Visualize the domain by creating a plot.

Parameters `viewer_app` : Viewer, optional

define the viewer to plot the domain default is viewer.matplotlibViewer

view_distance : boolean or int or list of int, optional

view the distance between the interior points and the border default is False

view_in : boolean, optional

view the inner points default is True

view_out : boolean, optional

view the outer points default is True

view_bound : boolean, optional

view the points on the bounds default is False

label : int or list of int, optional

view the distance only for the specified labels

Returns a figure representing the domain

the module storage

<code>Array(nv, gspace_size, vmax[, sorder, ...])</code>	This class defines the storage of the moments and distribution functions in pyLBM.
<code>SOA(nv, gspace_size, vmax, mpi_topo[, ...])</code>	This class defines a structure of arrays to store the unknowns of the lattice Boltzmann schemes.
<code>AOS(nv, gspace_size, vmax, mpi_topo[, ...])</code>	This class defines an array of structures to store the unknowns of the lattice Boltzmann schemes.

pyLBM.storage.Array

class pyLBM.storage.**Array** (*nv, gspace_size, vmax, sorder=None, mpi_topo=None, dtype=<type 'numpy.float64'>, gpu_support=False*)

This class defines the storage of the moments and distribution functions in pyLBM.

It sets the storage in memory and how to access.

Parameters **nv: int**

number of velocities

gspace_size: list of int

number of points in each direction including the fictitious point

vmax: list of int

the size of the fictitious points in each direction

sorder: list of int

the order of nv, nx, ny and nz Default is None which mean [nv, nx, ny, nz]

mpi_topo:

the mpi topology

dtype: type

the type of the array. Default is numpy.double

Attributes

<code>nspace</code>	the space size.
<code>nv</code>	the number of velocities.
<code>shape</code>	the shape of the array that stores the data.
<code>size</code>	the size of the array that stores the data.

pyLBM.storage.Array.nspace

`Array.nspace`
the space size.

pyLBM.storage.Array.nv

`Array.nv`
the number of velocities.

pyLBM.storage.Array.shape

`Array.shape`

the shape of the array that stores the data.

pyLBM.storage.Array.size

`Array.size`

the size of the array that stores the data.

array	
-------	--

Methods

<code>generate()</code>	generate periodic conditions functions for loo.py back-end.
<code>set_conserved_moments(consm, nv_ptr)</code>	add conserved moments information to have a direct access.
<code>update()</code>	update ghost points on the interface with the datas of the neighbors.

pyLBM.storage.Array.generate

`Array.generate()`

generate periodic conditions functions for loo.py backend.

pyLBM.storage.Array.set_conserved_moments

`Array.set_conserved_moments(consm, nv_ptr)`

add conserved moments information to have a direct access.

Parameters `consm` : dict

set the name and the location of the conserved moments. The format is

key : the conserved moment (sympy symbol or string)

value : list of 2 integers

first item : the scheme number

second item : the index of the conserved moment in this scheme

`nv_ptr` : list of int

store the location of the schemes

pyLBM.storage.Array.update

`Array.update()`

update ghost points on the interface with the datas of the neighbors.

pyLBM.storage.SOA

```
class pyLBM.storage.SOA(nv, gspace_size, vmax, mpi_topo, dtype=<type 'numpy.float64'>,
                        gpu_support=False)
```

This class defines a structure of arrays to store the unknowns of the lattice Boltzmann schemes.

Parameters **nv: int**

number of velocities

gspace_size: list of int

number of points in each direction including the fictitious point

vmax: list of int

the size of the fictitious points in each direction

mpi_topo:

the mpi topology

dtype: type

the type of the array. Default is numpy.double

Attributes

<i>nspace</i>	the space size.
<i>nv</i>	the number of velocities.
<i>shape</i>	the shape of the array that stores the data.
<i>size</i>	the size of the array that stores the data.

pyLBM.storage.SOA.nspace

SOA.nspace

the space size.

pyLBM.storage.SOA.nv

SOA.nv

the number of velocities.

pyLBM.storage.SOA.shape

SOA.shape

the shape of the array that stores the data.

pyLBM.storage.SOA.size

SOA.size

the size of the array that stores the data.

array

Methods

<code>generate()</code>	generate periodic conditions functions for loo.py back-end.
<code>reshape()</code>	reshape.
<code>set_conserved_moments(consm, nv_ptr)</code>	add conserved moments information to have a direct access.
<code>update()</code>	update ghost points on the interface with the datas of the neighbors.

pyLBM.storage.SOA.generate

`SOA.generate()`
generate periodic conditions functions for loo.py backend.

pyLBM.storage.SOA.reshape

`SOA.reshape()`
reshape.

pyLBM.storage.SOA.set_conserved_moments

`SOA.set_conserved_moments (consm, nv_ptr)`
add conserved moments information to have a direct access.

Parameters `consm` : dict

set the name and the location of the conserved moments. The format is

key : the conserved moment (sympy symbol or string)

value : list of 2 integers

first item : the scheme number

second item : the index of the conserved moment in this scheme

`nv_ptr` : list of int

store the location of the schemes

pyLBM.storage.SOA.update

`SOA.update()`
update ghost points on the interface with the datas of the neighbors.

pyLBM.storage.AOS

`class pyLBM.storage.AOS (nv, gspace_size, vmax, mpi_topo, dtype=<type 'numpy.float64'>, gpu_support=False)`

This class defines an array of structures to store the unknowns of the lattice Boltzmann schemes.

Parameters `nv`: int

number of velocities

gspace_size: list of int

number of points in each direction including the fictitious point

vmax: list of int

the size of the fictitious points in each direction

mpi_topo:

the mpi topology

dtype: type

the type of the array. Default is numpy.double

Attributes

<i>nspace</i>	the space size.
<i>nv</i>	the number of velocities.
<i>shape</i>	the shape of the array that stores the data.
<i>size</i>	the size of the array that stores the data.

pyLBM.storage.AOS.nspace

AOS.**nspace**

the space size.

pyLBM.storage.AOS.nv

AOS.**nv**

the number of velocities.

pyLBM.storage.AOS.shape

AOS.**shape**

the shape of the array that stores the data.

pyLBM.storage.AOS.size

AOS.**size**

the size of the array that stores the data.

array

Methods

<i>generate()</i>	generate periodic conditions functions for loo.py back-end.
-------------------	---

Continued on next page

Table 2.37 – continued from previous page

<i>reshape()</i>	
<i>set_conserved_moments</i> (consm, nv_ptr)	add conserved moments information to have a direct access.
<i>update()</i>	update ghost points on the interface with the datas of the neighbors.

pyLBM.storage.AOS.generate

AOS.generate()
generate periodic conditions functions for loo.py backend.

pyLBM.storage.AOS.reshape

AOS.reshape()

pyLBM.storage.AOS.set_conserved_moments

AOS.set_conserved_moments (consm, nv_ptr)
add conserved moments information to have a direct access.

Parameters **consm** : dict

set the name and the location of the conserved moments. The format is

key : the conserved moment (sympy symbol or string)

value : list of 2 integers

first item : the scheme number

second item : the index of the conserved moment in this scheme

nv_ptr : list of int

store the location of the schemes

pyLBM.storage.AOS.update

AOS.update()
update ghost points on the interface with the datas of the neighbors.

the module bounds

The module bounds contains the classes needed to treat the boundary conditions with the LBM formalism

The classes are

<i>Boundary</i> (domain, dico)	Construct the boundary problem by defining the list of indices on the border and the methods used on each label.
<i>Boundary_method</i> (istore, ilabel, distance, ...)	Set boundary method.
<i>bounce_back</i> (istore, ilabel, distance, ...)	Boundary condition of type bounce-back
<i>anti_bounce_back</i> (istore, ilabel, distance, ...)	Boundary condition of type anti bounce-back
Continued on next page	

Table 2.38 – continued from previous page

Neumann(istore, ilabel, distance, stencil, ...)

Boundary condition of type Neumann

pyLBM.boundary.Boundary

class pyLBM.boundary.**Boundary** (*domain, dico*)

Construct the boundary problem by defining the list of indices on the border and the methods used on each label.

Parameters **domain** : Domain class**dico** : a dictionary that describes the boundaries

- key is a label
- value are again a dictionary with
 - “method” key that gives the boundary method class used (Bounce_back, Anti_bounce_back, ...)
 - “value_bc” key that gives the value on the boundary

Attributes

bv	(dictionary) for each label key, a list of spatial indices and distance define for each velocity the points on the domain that are on the boundary.
meth- ods	(list) list of boundary methods used in the LBM scheme The list contains Boundary_method instance.

pyLBM.boundary.Boundary_method

class pyLBM.boundary.**Boundary_method** (*istore, ilabel, distance, stencil, value_bc, nspace, backend*)

Set boundary method.

Parameters **None**

Attributes

feq	(NumPy array) the equilibrium values of the distribution function on the border
rhs	(NumPy array) the additional terms to fix the boundary values
distance	(NumPy array) distance to the border (needed for Bouzidi type conditions)
istore	(NumPy array)
ilabel	(NumPy array)
iload	(list)
value_bc	(dictionary) the prescribed values on the border

Methods

<i>fix_ilocad</i> ()	Transpose iload and istore.
<i>move2gpu</i> ()	Move arrays needed to compute the boundary on the GPU memory.

Continued on next page

Table 2.39 – continued from previous page

<code>prepare_rhs(simulation)</code>	Compute the distribution function at the equilibrium with the value on the border.
<code>update(ff)</code>	Update distribution functions with this boundary condition.

pyLBM.boundary.Boundary_method.fix_ilo

`Boundary_method.fix_ilo()`

Transpose ilo and istore.

Must be fix in a future version.

pyLBM.boundary.Boundary_method.move2gpu

`Boundary_method.move2gpu()`

Move arrays needed to compute the boundary on the GPU memory.

pyLBM.boundary.Boundary_method.prepare_rhs

`Boundary_method.prepare_rhs(simulation)`

Compute the distribution function at the equilibrium with the value on the border.

Parameters `simulation` : simulation class

pyLBM.boundary.Boundary_method.update

`Boundary_method.update(ff)`

Update distribution functions with this boundary condition.

Parameters `ff`

The distribution functions

pyLBM.boundary.bounce_back

`class pyLBM.boundary.bounce_back(istore, ilabel, distance, stencil, value_bc, nspace, backend)`

Boundary condition of type bounce-back

Notes

```
from __future__ import print_function, division
from six.moves import range

import numpy as np
import matplotlib.pyplot as plt
from matplotlib.patches import Rectangle

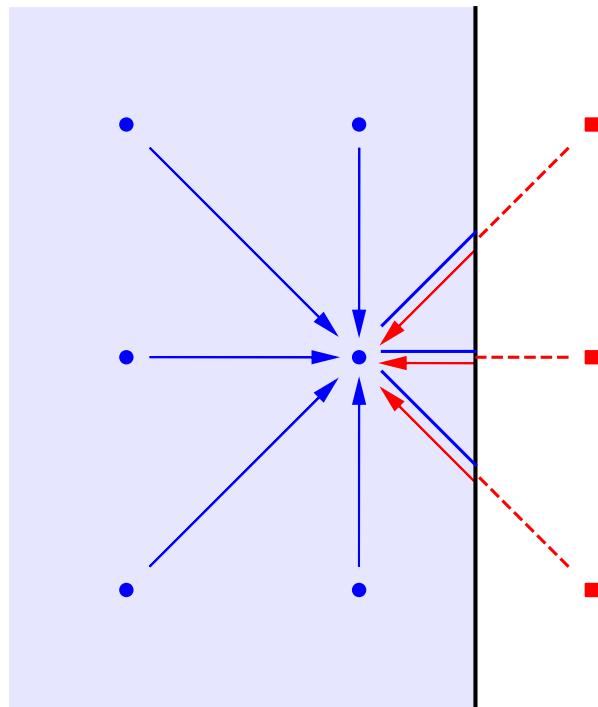
L = 2
H = 2
color_in = 'b'
color_out = 'r'
```

```

fig = plt.figure(1, figsize=(8, 8), facecolor='white')
ax = fig.add_subplot(111, aspect='equal')
ax.plot([0, 0], [-H, H-1], 'k-', linewidth = 2)
ax.add_patch(Rectangle((0, -H), -L, 2*H-1, alpha=0.1, fill=True, color=color_in))
# inner points
mesh_x = np.arange(-L,0) + 0.5
mesh_y = np.arange(-H,H-1) + 0.5
mesh_Y, mesh_X = np.meshgrid(mesh_y, mesh_x)
ax.scatter(mesh_X, mesh_Y, marker='o', color=color_in)
# outer points
mesh_x = np.arange(0,L-1) + 0.5
mesh_y = np.arange(-H,H-1) + 0.5
mesh_Y, mesh_X = np.meshgrid(mesh_y, mesh_x)
ax.scatter(mesh_X, mesh_Y, marker='s', color=color_out)
# inner arrows
e = 0.1
x, y = -0.5, -0.5
for i in [-1,0]:
    for j in [-1,0,1]:
        if i != 0 or j != 0:
            ax.arrow(x+i*(1-e), y+j*(1-e), -i*(1-2*e), -j*(1-2*e),
                    length_includes_head=True,
                    head_width=.5*e,
                    head_length=e,
                    fc=color_in,
                    ec=color_in)
# outer arrows
for j in [-1,0,1]:
    vx = np.array([x+e, x+0.5])
    vy = np.array([y+j*e, y+j*.5])
    ax.plot(vx, vy+.25*(1+.5*abs(j))*e, c=color_in)
    ax.arrow(vx[1], vy[1]-.25*(1+.5*abs(j))*e, -0.5+e, j*(e-.5),
            length_includes_head=True,
            head_width=.5*e,
            head_length=e,
            fc=color_out,
            ec=color_out)
    ax.plot([x+1-e, x+0.5], [y+(1-e)*j,y+.5*j], c=color_out, linestyle='--')
ax.axis('off')
plt.title("bounce back: the exiting particles bounce back without sign_
↪modification")
plt.show()

```

bounce back: the exiting particles bounce back without sign modification



Methods

<code>fix_iloader()</code>	Transpose iload and istore.
<code>generate(sorder)</code>	Generate the numerical code.
<code>move2gpu()</code>	Move arrays needed to compute the boundary on the GPU memory.
<code>prepare_rhs(simulation)</code>	Compute the distribution function at the equilibrium with the value on the border.
<code>set_iloader()</code>	Compute the indices that are needed (symmetric velocities and space indices).
<code>set_rhs()</code>	Compute and set the additional terms to fix the boundary values.

Continued on next page

Table 2.40 – continued from previous page

<code>update(ff)</code>	Update distribution functions with this boundary condition.
-------------------------	---

pyLBM.boundary.bounce_back.fix_ilo

`bounce_back.fix_ilo()`

Transpose ilo and istore.

Must be fix in a future version.

pyLBM.boundary.bounce_back.generate

`bounce_back.generate(sorder)`

Generate the numerical code.

Parameters `sorder` : list of int

the order of nv, nx, ny and nz

pyLBM.boundary.bounce_back.move2gpu

`bounce_back.move2gpu()`

Move arrays needed to compute the boundary on the GPU memory.

pyLBM.boundary.bounce_back.prepare_rhs

`bounce_back.prepare_rhs(simulation)`

Compute the distribution function at the equilibrium with the value on the border.

Parameters `simulation` : simulation class

pyLBM.boundary.bounce_back.set_ilo

`bounce_back.set_ilo()`

Compute the indices that are needed (symmetric velocities and space indices).

pyLBM.boundary.bounce_back.set_rhs

`bounce_back.set_rhs()`

Compute and set the additional terms to fix the boundary values.

pyLBM.boundary.bounce_back.update

`bounce_back.update(ff)`

Update distribution functions with this boundary condition.

Parameters `ff`

The distribution functions

pyLBM.boundary.anti_bounce_back

`class pyLBM.boundary.anti_bounce_back` (*istore, ilabel, distance, stencil, value_bc, nspace, backend*)
Boundary condition of type anti bounce-back

Notes

```
from __future__ import print_function, division
from six.moves import range

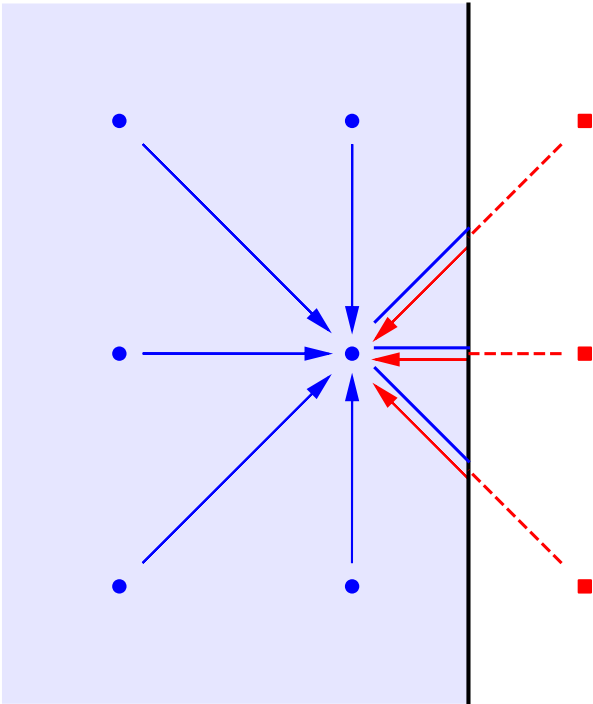
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.patches import Rectangle

L = 2
H = 2
color_in = 'b'
color_out = 'r'

fig = plt.figure(1, figsize=(8, 8), facecolor='white')
ax = fig.add_subplot(111, aspect='equal')
ax.plot([0, 0], [-H, H-1], 'k-', linewidth = 2)
ax.add_patch(Rectangle((0, -H), -L, 2*H-1, alpha=0.1, fill=True, color=color_in))
# inner points
mesh_x = np.arange(-L, 0) + 0.5
mesh_y = np.arange(-H, H-1) + 0.5
mesh_Y, mesh_X = np.meshgrid(mesh_y, mesh_x)
ax.scatter(mesh_X, mesh_Y, marker='o', color=color_in)
# outer points
mesh_x = np.arange(0, L-1) + 0.5
mesh_y = np.arange(-H, H-1) + 0.5
mesh_Y, mesh_X = np.meshgrid(mesh_y, mesh_x)
ax.scatter(mesh_X, mesh_Y, marker='s', color=color_out)
# inner arrows
e = 0.1
x, y = -0.5, -0.5
for i in [-1, 0]:
    for j in [-1, 0, 1]:
        if i != 0 or j != 0:
            ax.arrow(x+i*(1-e), y+j*(1-e), -i*(1-2*e), -j*(1-2*e),
                    length_includes_head=True,
                    head_width=.5*e,
                    head_length=e,
                    fc=color_in,
                    ec=color_in)
# outer arrows
for j in [-1, 0, 1]:
    vx = np.array([x+e, x+0.5])
    vy = np.array([y+j*e, y+j*.5])
    ax.plot(vx, vy+.25*(1+.5*abs(j))*e, c=color_in)
    ax.arrow(vx[1], vy[1]-.25*(1+.5*abs(j))*e, -0.5+e, j*(e-.5),
            length_includes_head=True,
            head_width=.5*e,
            head_length=e,
            fc=color_out,
            ec=color_out)
```

```
ax.plot([x+1-e, x+0.5], [y+(1-e)*j,y+.5*j], c=color_out, linestyle='--')
ax.axis('off')
plt.title("anti bounce back: the exiting particles bounce back with sign_
↔modification")
plt.show()
```

anti bounce back: the exiting particles bounce back with sign modification



Methods

<code>fix_ilo</code>	Transpose ilo and istore.
<code>generate(sorder)</code>	Generate the numerical code.
<code>move2gpu()</code>	Move arrays needed to compute the boundary on the GPU memory.

Continued on next page

Table 2.41 – continued from previous page

<code>prepare_rhs(simulation)</code>	Compute the distribution function at the equilibrium with the value on the border.
<code>set_iloading()</code>	Compute the indices that are needed (symmetric velocities and space indices).
<code>set_rhs()</code>	Compute and set the additional terms to fix the boundary values.
<code>update(ff)</code>	Update distribution functions with this boundary condition.

pyLBM.boundary.anti_bounce_back.fix_iloading

`anti_bounce_back.fix_iloading()`

Transpose iloading and istore.

Must be fix in a future version.

pyLBM.boundary.anti_bounce_back.generate

`anti_bounce_back.generate(sorder)`

Generate the numerical code.

Parameters `sorder` : list of int

the order of nv, nx, ny and nz

pyLBM.boundary.anti_bounce_back.move2gpu

`anti_bounce_back.move2gpu()`

Move arrays needed to compute the boundary on the GPU memory.

pyLBM.boundary.anti_bounce_back.prepare_rhs

`anti_bounce_back.prepare_rhs(simulation)`

Compute the distribution function at the equilibrium with the value on the border.

Parameters `simulation` : simulation class

pyLBM.boundary.anti_bounce_back.set_iloading

`anti_bounce_back.set_iloading()`

Compute the indices that are needed (symmetric velocities and space indices).

pyLBM.boundary.anti_bounce_back.set_rhs

`anti_bounce_back.set_rhs()`

Compute and set the additional terms to fix the boundary values.

pyLBM.boundary.anti_bounce_back.update

`anti_bounce_back.update(ff)`

Update distribution functions with this boundary condition.

Parameters ff

The distribution functions

pyLBM.boundary.Neumann

class `pyLBM.boundary.Neumann` (*istore, ilabel, distance, stencil, value_bc, nspace, backend*)

Boundary condition of type Neumann

Methods

<code>fix_iloading()</code>	Transpose iload and istore.
<code>generate(sorder)</code>	Generate the numerical code.
<code>move2gpu()</code>	Move arrays needed to compute the boundary on the GPU memory.
<code>prepare_rhs(simulation)</code>	Compute the distribution function at the equilibrium with the value on the border.
<code>set_iloading()</code>	Compute the indices that are needed (symmetric velocities and space indices).
<code>set_rhs()</code>	Compute and set the additional terms to fix the boundary values.
<code>update(ff)</code>	Update distribution functions with this boundary condition.

pyLBM.boundary.Neumann.fix_iloading

`Neumann.fix_iloading()`

Transpose iload and istore.

Must be fix in a future version.

pyLBM.boundary.Neumann.generate

`Neumann.generate(sorder)`

Generate the numerical code.

Parameters sorder : list of int

the order of nv, nx, ny and nz

pyLBM.boundary.Neumann.move2gpu

`Neumann.move2gpu()`

Move arrays needed to compute the boundary on the GPU memory.

pyLBM.boundary.Neumann.prepare_rhs

`Neumann.prepare_rhs (simulation)`

Compute the distribution function at the equilibrium with the value on the border.

Parameters `simulation` : simulation class

pyLBM.boundary.Neumann.set_iloal

`Neumann.set_iloal ()`

Compute the indices that are needed (symmetric velocities and space indices).

pyLBM.boundary.Neumann.set_rhs

`Neumann.set_rhs ()`

Compute and set the additional terms to fix the boundary values.

pyLBM.boundary.Neumann.update

`Neumann.update (ff)`

Update distribution functions with this boundary condition.

Parameters `ff`

The distribution functions

CHAPTER 3

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CHAPTER 4

Indices and tables

- `genindex`
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